UNIVERSITY OF ŽILINA IN ŽILINA
FACULTY OF MANAGEMENT SCIENCE AND INFORMATICS

TIME-DEPENDENT ANALYSIS OF SYSTEM RELIABILITY BASED ON LOGIC DIFFERENTIAL CALCULUS

Dissertation thesis

Registration number: 28360020203006

Study program:	Applied Informatics
Field of study:	Informatics
Workplace:	Department of Informatics
	Faculty of Management Science and Informatics, University of Žilina
Supervisor:	Prof. Ing. Elena Zaitseva, PhD.

Žilina, 2020 **Ing. Patrik Rusnák**

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ABSTRAKT

Rusnák, Patrik: Časovo závislá analýza spoľahlivosti systémov na základe použitia logického diferenciálneho počtu. [Dizertačná práca]

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Skúmanie spoľahlivosti systému je komplexný problém, ktorý zahŕňa mnoho úloh. Jednou z takýchto úloh je hodnotenie dôležitosti systémových komponentov. Tieto informácie môžu byť použité pre rôzne účely, ako napríklad pri údržbe systému alebo pri optimalizácii spoľahlivosti systému. Hlavným cieľom tejto práce je vývoj nových prístupov pre analýzu spoľahlivosti systému založenej na štruktúrnej funkcii, ktoré umožnia časovo závislú analýzu systému a jeho komponentov a tiež ktoré riešia problém reprezentácie systému s veľkým počtom komponentov. Prvá časť tohto cieľa bude riešená použitím logického diferenciálneho počtu a druhá časť bude riešená pomocou podpisu prežitia pre reprezentáciu systému. Tiež budú tieto prístupy demonštrované na vybraných prípadových štúdiách.

Kľúčové slová: Štruktúrna funkcia, logický diferenciálny počet, podpis prežitia, analýza spoľahlivosti, indexy dôležitosti;

ABSTRACT

Examining system reliability is a complex problem that involves many tasks. One such task is to evaluate the importance of system components. This information can be used for a variety of purposes, such as system maintenance or optimizing system reliability. The principal goal of this work is to develop new approaches for reliability analysis of system based on the structure function that allows time-dependent analysis of the system and to reduce the mathematical representation of system with large number of components. The first part of this goal will be solved using a logical differential calculus and the second part will be solved using a survival signature. These new approaches will also be demonstrated in selected case studies.

Key words: structure function, logical differential calculus, survival signature, reliability analysis, importance measures;

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Used Acronyms

BSS	Binary-State System
MSS	Multi-State System
HDD	Hard Disk Drive
RAID	Redundant Array of Inexpensive (or Independent) Disks
RBD	Reliability block diagram
IID	Independent and identically distributed
XOR	Exclusive Or
DPLD	Direct Partial Logic Derivative
IM	Importance measure
SI	Structure Importance
BI	Birnbaum’s Importance
CI	Criticality Importance
JRI	Joint Reliability Importance
FI	Failure Importance
MTTF	Mean time to failure
AFR	Annualized Failure Rate

Used Notations

x_i - Boolean variable, which represents the state of the i-th component;

$\boldsymbol{x}$ - state vector that contains information about the state of all system components;

n - number of system components;

ϕ - structure function;

$\wedge$ - Boolean operation AND;

$\vee$ - Boolean operation OR;

$\oplus$ - Boolean operation XOR;

$\overline{}$ - Boolean operation NOT;

$z(t)$ - system state function at time t;

$Z(t)$ - random variable modelling behavior of the system at time t;

X - random variable that takes value from set $\{0,1\}$;

$x_i(t)$ - function that defines state of the i-th component at time t;

$\boldsymbol{x}(t)$ - vector that contains $x_i(t)$ for each system components;

A - system availability;

U- system unavailability;

p_i - probability that the i-th component will work during its mission time;

$\boldsymbol{p}$ - vector that contains probabilities p_i for each system components;

q_i - probability that the i-th component will not work during its mission time;

$\boldsymbol{q}$ - vector that contains probabilities q_i for each system components;

$A(t)$ - system availability function at time t;

$U(t)$ - system unavailability function at time t;

$P_i(t)$ - probability that the i-th component will work at time t given it is working at time 0;

$\boldsymbol{P}(t)$ - vector that contains $P_i(t)$ for each system components;

$Q_i(t)$ - probability that the i-th component fails no later than at time t given it is working at time 0;

$\boldsymbol{Q}(t)$ - vector that contains $Q_i(t)$ for each system components;

R - system reliability;

F - system unreliability;

$R(t)$ - system reliability function at time t;

$F(t)$ - system unreliability function at time t;

$\frac{\partial\phi(\boldsymbol{x})}{\partial x_i}$ – logic derivation of the Boolean function ϕ with respect to variable x_i;

$\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}, \frac{\partial\phi(0\to 1)}{\partial x_i(0\to 1)}$ - direct partial logic derivative of the Boolean function ϕ with respect to variable x_i;

$\frac{\partial\phi(1\to 0)}{\partial x_i(0\to 1)}, \frac{\partial\phi(0\to 1)}{\partial x_i(1\to 0)}$ - inverse partial logic derivative of the Boolean function ϕ with respect to variable x_i;

$\frac{\partial\phi(1\to 0)}{\partial (x_i,x_j,\ldots)(1,1,\ldots)\to(0,0,\ldots)}$ - direct partial logic derivative of the Boolean function ϕ with respect to vector of variables $(x_i, x_j, \ldots)$;

$\mathbf{s}_\tau$ - system signature of the system τ;

K - number of different types of the system components;

n_k - number of system components of type k;

$\boldsymbol{x}^k$ - state vector that contains information about the state of all system components of type k;

Φ - survival signature;

l, l_k - number of the working system components and system components of type k;

S_l , S_l^k - set that contains all state vectors $\boldsymbol{x}, \boldsymbol{x}^k$, in which exactly l system components or l system components of type k are working;

$S_{l_1,\ldots,l_K}$ - set that contains all state vectors $\boldsymbol{x}$, in which exactly l_k components of type k are working for each $k = 1, \ldots, K$;

C, C^k - number of components or components of type k in the system that are in working state;

C_t, C_t^k - number of components or components of type k in the system that are in working state at specified time point $t > 0$;

SI_i - structure importance of the i-th component;

$\mathrm{TD}(.)$ - truth density of the argument interpreted as a Boolean function;

BI_i - Birnbaum's importance of the i-th component;

CI_i - criticality importance of the i-th component;

$\mathrm{JRI}_{i,j}$ - joint reliability importance of the i-th and j-th component;

$\mathrm{FI}_{i,j}$ - failure importance of the i-th and j-th component;

$\mathrm{BI}_i(t)$ - time-dependent Birnbaum's importance of the i-th component;

$\mathrm{CI}_i(t)$ - time-dependent criticality importance of the i-th component;

$\mathrm{JRI}_{i,j}(t)$ - time-dependent joint reliability importance of the i-th and j-th component;

$\mathrm{FI}_{i,j}(t)$ - time-dependent failure importance of the i-th and j-th component;

$\frac{\partial \Phi(l_1,\ldots,l_K)\downarrow}{\partial l_k(a\to a-1)}$ - first direct partial logic derivative of the survive signature Φ with respect to type l_k;

$\mathrm{SI}_{k,a}^{\downarrow}$ - structure importance for a working components of type k that uses first direct partial logic derivative;

$\frac{\partial \Phi(l_1,\ldots,l_K)\downarrow}{\partial l_k\downarrow}$ - second direct partial logic derivative of the survive signature Φ with respect to type l_k;

$\mathrm{SI}_k^{\downarrow}$ - structure importance for components of type k that uses second direct partial logic derivative;

$\frac{\partial \Phi(l_1,\ldots,l_K)\Downarrow}{\partial l_k\downarrow}$ - third direct partial logic derivative of the survive signature Φ with respect to type l_k;

$\mathrm{SI}_k^{\Downarrow}$ - structure importance for components of type k that uses third direct partial logic derivative;

Introduction

Reliability engineering is a multidisciplinary scientific field that provides the methods necessary to quantify the reliability of the system, to test the design of the system, to analyse the system and its components, etc. The main problems of reliability engineering can be defined as follows [1]:

- application of the theoretical knowledge and mathematical techniques to prevent or reduce the likelihood of failure occurrence;
- identification and solving the causes of failures that occur in system despite failure prevention;
- definition of processes, which will manage failures that may occur if the causes of these failures have not been resolved;
- application of the methods to estimate the reliability of new system design and to analyse reliability data.

Important step in reliability evaluation of system is the development of its mathematical representation. As has been shown in [1], this mathematical representation must allow investigating the system failure, e.g., mechanisms of failure and its consequences; measuring system reliability; analysing critical states of system reliability; elaborating maintenance of the system, fault diagnosis and prognosis.

The most often used mathematical representation of a system in reliability analysis is a model that takes into account two important states of system: failure and working state. This mathematical model is known as Binary-State System (BSS) that has been introduced as one of the first [2]–[4] . This will also be used in this work. Boolean logic is mainly used for analysis of systems represented as BSS. Another used mathematical representation of a system is known as Multi-State System (MSS). This mathematical representation allows to perform reliability analysis with more than two performance levels and is used to define multiple states for the system and its components and to perform a more detailed reliability analysis of the states of the system or its components [5], [6].

There are various methods to evaluate the system reliability and failure based on these mathematical models. All these methods can be divided into four groups depending on the mathematical background [1], [3]: methods based on structure function, stochastic methods, Monte-Carlo simulation and methods based on universal generation function. The structure function based methods permit mathematically representing system of any structural complexity [2] and will be used in this work.

The structure function defines univalent correlation of a system performance level on components states and is used to represent system composed of *n* components [7]. The BSS structure function can be viewed as a Boolean function, which can be easily used in reliability analysis of the steady-state system [7], [8]. Such a mathematical representation is time-independent. Important advantage of this representation is possibility to use the well-developed and useful mathematical approach of Boolean algebra in reliability evaluation of the investigated system. Effective methods in reliability analysis were developed with application of Boolean algebra for minimal cut/path sets definition [8], frequency characteristics of system reliability [7], or importance measures calculation [9]. The structure function has its relevant role in modern development in reliability analysis, for example, in case of multi-function system reliability [10], general multilinear expression of the structure function of an arbitrary semi-coherent system [11], or Graphs models and algorithms for reliability assessment [12]. The disadvantage of these methods is analysis of system in stationary state. On the other hand, the structure function in a form of Boolean function can be used for calculation of the reliability function of the system that represents the probability of the system to be in functioning state during its mission time or specific time. In this case, special methods for this calculation should be developed [3]. Although, reliability function is important in reliability analysis, it is not sufficient to give a complete picture about system reliability. Another necessary constituent of reliability evaluation is importance analysis. Methods for calculation of Importance Measures (IMs), which quantify influence of the system components on the whole system, based on application of system representation by the structure function and logic differential calculus have been considered in [13], [14] for a system in stationary state.

The structure function can be formed simply for a system with any structural complexity. At the same time, the structure function dimension will increases significantly with increasing number of system components. The evaluation of the structure function is complicated for the systems with large number of components, when the uncertainty of components behaviour is taking into account. This is important because the information for the quantitative specification of the uncertainties associated with the components is often limited and appears as incomplete information. Therefore, the methods for the dimensional reduction should be developed for the structure function. A possible way is application of modern approach known as survival signature [15], which focuses on system survivability with system with K types of components [16].

In reliability analysis, the concept of the system signature has been recognized as an important tool to quantify the reliability of systems with or without time consideration. Specific of these investigation is analysis and evaluation of systems that are formed by more than one type of components [15]. Recent advancements using the concept of system signature in reliability analysis are reported in [17]–[20]. Typically, the system signature is associated with the assumption that all components in the system are of the same type that is limitation for real systems. The system signature with different types of components has been considered in [15] as system survival signature. This approach has been further developed for the purposes of reliability analysis in [17], [18]. In paper [19], this approach has been developed for importance analysis of system. This method is effective, but is computational demanding. Typically, the methods for importance analysis are based on the different mathematical methodologies [14]. One such methodology is logic differential calculus [21], [22] that is standardly used for analysing of the system represented by the structure function and not for survival signature.

By taking all the previously mentioned information into account, the structure function in form of Boolean function is simple mathematical representation of a system in reliability analysis which can be formed for a system of any structural complexity and evaluated based on well-developed methods related to Boolean functions. The disadvantages of this mathematical representation is (a) high dimension for system with large number of components and (b) impossibility of time-depend analysis. *The principal goal of this work is to develop new approaches for reliability analysis of system based on the structure function that allows time-dependent analysis of the system and to reduce the dimension of mathematical representation of system with large number of components*. This goal can be divided to two parts. The first part can be solved by developing of the new approach for time-dependent importance analysis of the system mathematically represented by the structure function that is based on the logic differential calculus. The second part can be solved by the use of mathematical approach of survival signature for the system representation. To achieve the principal goal, the following tasks are addressed in this work:

- continuation in investigation from [13], [14] and proposing an approach for system importance analysis depending on time and based on the application of logic differential calculus in Boolean algebra;
- showing the efficiency and usability of this approach on selected systems;
- defining the extension of direct partial logic derivatives (DPLDs) for analysis of system dynamic properties based on system survival signature [15];
- illustration of the use of proposed approach of direct partial logical derivatives for system signature on selected systems.

This work is divided into three chapters. In the first chapter, we will describe basic terms in reliability analysis using Boolean logic such as structure function, reliability, unreliability, system signature, survival signature and logic differential calculus, especially direct partial logic derivatives.

In the first part of the second chapter, we will show how IMs can be calculated and how the logic differential calculus can be used for their calculation. Then, we will present a new approach to compute time-dependent IMs based on structure function and logic differential calculus. Finally, we will introduce the new IM based on logic differential calculus that focuses on the analysis of the pair of the system components. In the next part of the second chapter, the new DPLDs and SI measures for survival signature will be shown.

Lastly, in the third chapter, we will demonstrate the usage of the new approaches in reliability analysis of selected case studies such as hydro power plant, data storage system, or surveillance system.

1 Boolean Logic in Reliability Engineering

Reliability analysis is used to study the properties of analysed system and its components [1]. In this section, we are considering systems and its components with two states that can be interpreted as functioning and failure. In order to describe the analysed system, a mathematical approach that can represent activity of the system is needed [1], [4]. One approach is to represent the system using the structure function.

1.1 Structure Function

The structure function is a mapping that defines value of system state for each combination of states of the system components. If we assume that the system is composed of n components, then this mapping is as follows [3]:

$$\phi(x_1, x_2 \dots, x_n) = \phi(\boldsymbol{x}): \ \{0,1\}^n \rightarrow \{0,1\}, \tag{1.1}$$

where x_i is a variable that defines state of component i for $i = 1,2, \dots, n$ and $\boldsymbol{x} = (x_1, x_2, \dots, x_n)$ is a vector of states of the system components (state vector). For example, let us consider a data storage system consisting of two main modules, in which the same data is stored. In the first part, two hard drive disks (HDDs) are organized in Redundant Array of Independent Disks (RAID) 0. In RAID 0, the capacity of the unit is equal to the sum of capacities of the used drives, which implies no redundancy of data. Therefore, failure of one drive means that the entire RAID 0 is lost. In the second part, the single HDD is used to store data. At least one part must be in working state to write and read data successfully. This system can be seen in form of reliability block diagram (RBD) that is shown in Fig. 1.1. According to the system description, its structure function can be represented by the following logic expression:

$$\phi(x_1, x_2, x_3) = x_1 \wedge x_2 \vee x_3 \tag{1.2}$$

where the operator $\wedge$ represents the Boolean operation AND and operator $\vee$ represents the Boolean operation OR.

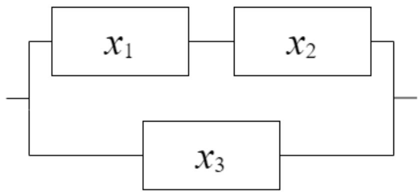


Fig. 1.1 Reliability block diagram of data storage system

From this point forward, we will assume that the analysed system is coherent, which means that structure function $\phi(\boldsymbol{x})$ is not decreasing in any of the variables and all the components are relevant for system operation [3], [14]. The data storage system is an example of the coherent system because its structure function (1.2) is not decreasing in any of the variables, and each component – HDD is needed for system operation. This means that there are not situations, in which the HDD failure will result in system functionality, if the system was failed.

Knowledge of the structure function allows us to investigate topological properties of the system. For example, we can use it to find the most reliable topology in a set of systems with different topologies [8] or evaluate importance of the components of a system and find those with the greatest influence on system operation from topological point of view [8], [13], [17], [23], [24]. However, its knowledge is not sufficient in performing time-dependent reliability analysis, which deals with evaluation of reliability of the system over time. For such purposes the system state function can be used.

System state at time t can be obtained from system state function $z(t)$ that has following form [3]:

$$z(t) = \phi\big(\boldsymbol{x}(t)\big) = \phi\big(x_1(t), x_2(t), \dots, x_n(t)\big): \langle 0, \infty) \rightarrow \{0,1\}, \tag{1.3}$$

where $x_i(t)$ for $i = 1,2, \dots, n$ is a function that defines state of the i-th component at time t. Although system state function $z(t)$ is closely related to structure function $\phi(\boldsymbol{x})$, these two functions are very different in their nature because the former is a function of time, while the latter is a function defining system topology, which is independent of time. If we consider the data storage system represented by (1.2), its state function has the following form:

$$\phi(x_1(t), x_2(t), x_3(t)) = x_1(t) \wedge x_2(t) \vee x_3(t). \tag{1.4}$$

Example of time courses for (1.4) and each component during specified time period in days can be seen in Fig. 1.2, where on the x-axis are values that represent number of days and on the y-axis is state (0 - failed, 1 - working). From time courses it is possible to see that the system will fail during 232th day, i.e. when the third component fails, which is caused by the fact, that the first and the second components failed and according to the (1.4) when the third component fails the system will fail as well. This is caused by system topology and by the fact that there are no repairs of the system or its components, which means that the data storage system is non-repairable during specified time period.

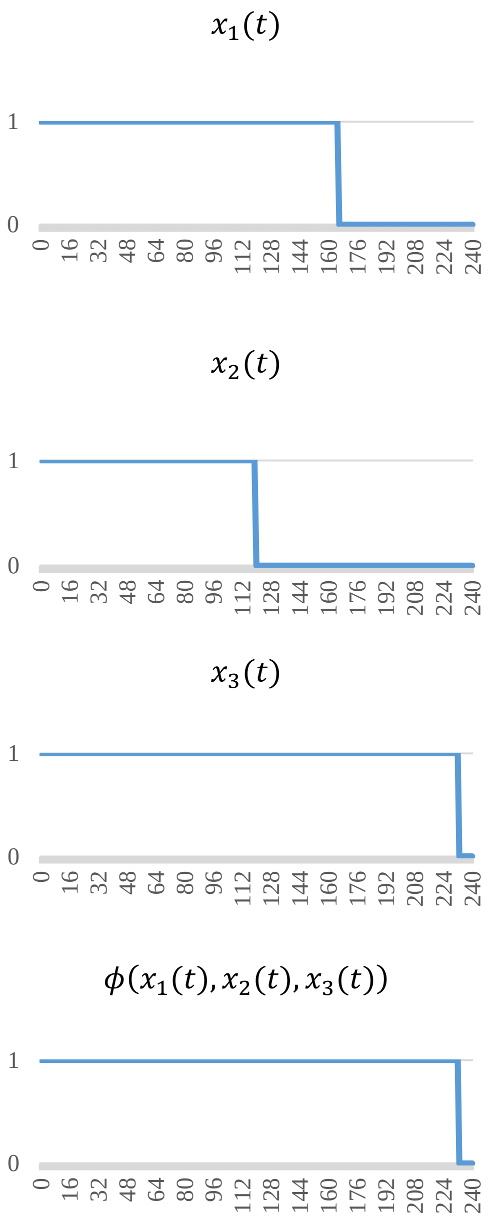


Fig. 1.2 State function of the data storage system and its components

The system state function can be viewed as a composition of system structure function and one specific realization of the state functions of all the system components, which means that the system state function $z(t)$ can also be viewed as one realization of uncountable many system state functions. This implies that evolution of the system over time can be viewed as the following stochastic process:

$$\{Z(t); t \geq 0\}, \tag{1.5}$$

where $Z(t)$ is a random variable modelling behavior of the system at time t.

Let us evaluate function $Z(t)$ at fixed time. In such a case, we obtain random variable X that takes value from set $\{0,1\}$ with probability A or U. These probabilities are known as system availability and unavailability, and they represent one of the basic reliability characteristic of a BSS [3]. In terms of single system component, those probabilities are p_i and q_i and are defined as follows [3]:

$$\begin{gathered} p_i = \Pr\{x_i = 1\}, q_i = \Pr\{x_i = 0\}, \\ p_i + q_i = 1. \end{gathered} \tag{1.6}$$

If we know random variable x_i, which models behavior of component i at fixed time, for each system component, i.e., for $i = 1,2, \dots, n$, and if we assume that the components are independent, then random variable X can be obtained by combining random variables x_i using the structure function. This allows us to compute the system state probabilities using the following formula [3]:

$$p = \Pr\{\phi(\boldsymbol{x}) = 1\}, q = \Pr\{\phi(\boldsymbol{x}) = 0\}, \tag{1.7}$$

where $\boldsymbol{x} = (x_1, x_2, \dots, x_n)$ is a vector of random variables modeling behavior of the system components at fixed time. This definition implies that the system availability A and unavailability U can be viewed as functions of component state probabilities [3]:

$$\begin{gathered} A = A(\boldsymbol{p}) = \Pr\{\phi(\boldsymbol{x}) = 1\}, U = U(\boldsymbol{q}) = \Pr\{\phi(\boldsymbol{x}) = 0\}, \\ A + U = 1, \end{gathered} \tag{1.8}$$

where $\boldsymbol{p} = (p_1, p_2, \dots, p_n)$ and $\boldsymbol{q} = (q_1, q_2, \dots, q_n)$ are vectors whose elements are the state probabilities of individual system components.

Formula (1.8) allows us to find the system state probabilities if we know the structure function of the system and the state probabilities of the components. It can be used to investigate how specific changes in state probabilities of one or more components influence the system state probabilities or other reliability measures [3], [14], but it does not allow us to perform dynamic (time-dependent) analysis of a BSS. For this task, random variable X has to be replaced by function $Z(t)$, which defines how properties of random variable X changes over time. In this case, the system availability $A(t)$ and unavailability $U(t)$ become functions of time, i.e.:

$$A(t) = A(\boldsymbol{P}(t)) = \Pr\{\phi(\boldsymbol{x}(t)) = 1\}, t \geq 0,$$
$$U(t) = U(\boldsymbol{Q}(t)) = \Pr\{\phi(\boldsymbol{x}(t)) = 0\}, t \geq 0, \quad (1.9)$$
$$A(t) + U(t) = 1, t \geq 0,$$

where $\boldsymbol{P}(t) = (P_1(t), P_2(t), \dots, P_n(t))$ and $\boldsymbol{Q}(t) = (Q_1(t), Q_2(t), \dots, Q_n(t))$ are vector-valued functions, whose elements are functions defining the state probabilities of individual system components over time, and $\boldsymbol{x}(t) = (x_1(t), x_2(t), \dots, x_n(t))$ is a vector of random variables modelling behaviour of the system components over time. This function can be used to find how reliability of the system or importance of the components change as time flows.

The most important result of previous formulae is that the system state probabilities can be viewed as a function of the components state probabilities combined using the structure function (static analysis based on (1.8)) or as a composition of functions defining the state probabilities of the system components over time (time-dependent analysis based on (1.9)) defined again by the structure function. This means if the system components are independent and we know the structure function of the system and the (time-dependent) state probabilities of the components, then we are able to find the (time-dependent) system state probabilities. As one can see, a BSS can be analysed with respect to time (dynamic analysis) or regardless of time (static analysis). This implies that reliability measures of a BSS might or might not depend on time.

In reliability analysis, it is also needed to compute the system reliability R that represents a probability that the system will be functioning during its mission time (period of time during which the system is required to operate properly). It is needed to point out, that the system reliability has same meaning as the system availability for unrepairable systems, which are the main focus of this work. Therefore, the system reliability is defined as follows [3], [14]:

$$R = R(\boldsymbol{p}) = \Pr\{\phi(\boldsymbol{x}) = 1\}, \tag{1.10}$$

where $\boldsymbol{p} = (p_1, p_2, \dots, p_n)$ is a vector of probabilities of components being functioning during the mission time and p_i is the probability that component i will be functioning during the mission time (it agrees with reliability of the component).

A complementary measure to system reliability is system unreliability, which agrees with the probability that the system will fail during the mission time [3], [14]:

$$F = F(\boldsymbol{q}) = \Pr\{\phi(\boldsymbol{x}) = 0\} = 1 - R(\boldsymbol{p}), \tag{1.11}$$

where $\boldsymbol{q} = (q_1, q_2, \dots, q_n)$ is a vector of unreliabilities of the components and $q_i = 1 - p_i$ is the probability of a failure of component i during the mission time (it agrees with unreliability of the component).

As an example, we will compute the R and F for the data storage system represented by (1.2). Thanks to the fact that the storage system has a parallel topology with serial topology with two HDDs in one branch and single HDD in another branch, the R for data storage system has following form:

$$R = p_1 * p_2 + p_3 - p_1 * p_2 * p_3. \tag{1.12}$$

In case of the F for the data storage system, it can be easily computed using (1.11), i.e. $F = 1 - R = q_1 * q_3 + q_2 * q_3 - q_1 * q_2 * q_3$. In this example, we will be working with same HDDs with $p = 0.8$. Therefore, the R and F of the data storage system is as follows: $R = 0.8 * 0.8 + 0.8 - 0.8 * 0.8 * 0.8 = 0.928$ and $F = 1 - 0.928 = 0.072$.

Definitions of system reliability (1.10) and unreliability (1.11) are computed for the whole mission time of the system, but they does not take the specific time values into account. Therefore, they allow us to compute reliability or unreliability of the system only for given values of reliabilities/unreliabilities of the components. If we want to find functions $R(t)$ and $F(t)$ that define time courses of system reliability and unreliability (occurrence of system failure), we have to replace vector $\boldsymbol{x}$ by its time-dependent version. Similarly, vector $\boldsymbol{p}$ of reliabilities of the components has to be replaced by $\boldsymbol{P}(t) = (P_1(t), P_2(t), \dots, P_n(t))$ and vector $\boldsymbol{q}$ of unreliabilities of the components by time-dependent vector $\boldsymbol{Q}(t) = (Q_1(t), Q_2(t), \dots, Q_n(t))$ [14]. In this case, $Q_i(t)$ is lifetime distribution of component i. This distribution defines the probability that the component fails no later than at time t given it is working at time 0. After finishing this process, we obtain functions $R(t)$ and $F(t)$ that are defined as follows:

$$R(t) = R\big(\boldsymbol{P}(t)\big) = \Pr\big\{\phi\big(\boldsymbol{x}(t)\big) = 1\big\}, \tag{1.13}$$

$$F(t) = F\big(\boldsymbol{Q}(t)\big) = \Pr\big\{\phi\big(\boldsymbol{x}(t)\big) = 0\big\} = 1 - R(t). \tag{1.14}$$

The procedure described above allows us to find reliability and unreliability (failure) function of the system if the structure function of the system is known, and we have information about lifetime distributions of all the system components. This proves that structure function, which is a static representation of the system (it defines system topology independent of time), can be used in time-dependent (dynamic) reliability analysis.

We will show how the $R(t)$ and $F(t)$ can be computed on the data storage system system represented by (1.2). We will assume that each HDD is independent and because all HDDs has the same type, they are also identically distributed. Furthermore, we will be working with exponential distribution with $\lambda = 1/5{,}000$ days as lifetime distribution of each HDD. According to (1.13) and (1.14), the $R(t)$ and $F(t)$ for data storage system has the following form:

$$R(t) = P_1(t) * P_2(t) + P_3(t) - P_1(t) * P_2(t) * P_3(t). \tag{1.15}$$

$$F(t) = 1 - R(t) = Q_1(t) * Q_3(t) + Q_2(t) * Q_3(t) - Q_1(t) * Q_2(t) * Q_3(t) \tag{1.16}$$

By using (1.15) and (1.16) it is possible to compute the values of $R(t)$ and $F(t)$. Their time courses for 10,000 days are depicted in Fig. 1.3, where $R(t)$ is shown as a blue solid line and $F(t)$ is shown as a red dotted line.

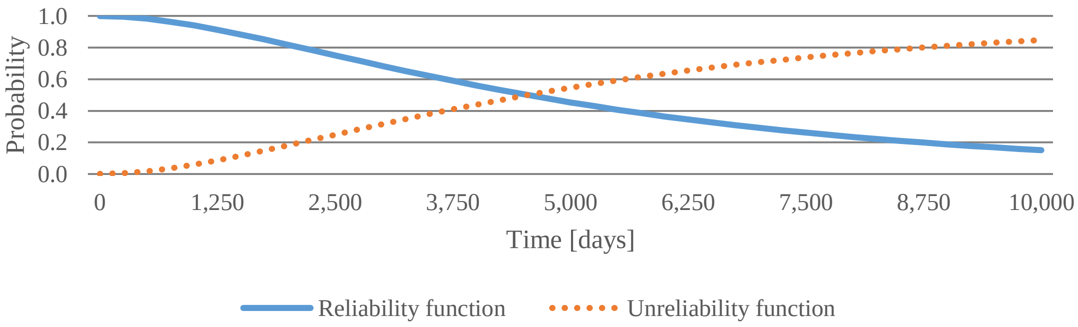


Fig. 1.3 Reliability and Unreliability function of the data storage system

1.2 Logic Differential Calculus

Definition (1.1) of the structure function corresponds to the definition of Boolean function [13]. This means it is possible to use mathematical methodology of Boolean algebra in reliability analysis based on structure function. One useful part of this methodology, which can be used to analyze how failure of a component affects the system operation, is logic differential calculus [25]. The central term of this methodology is logic derivative defined as follows [13], [25]:

$$\frac{\partial\phi(\boldsymbol{x})}{\partial x_i} = \phi(x_i, \boldsymbol{x}) \oplus \phi(\bar{x}_i, \boldsymbol{x}) = \phi(0_i, \boldsymbol{x}) \oplus \phi(1_i, \boldsymbol{x}), \tag{1.17}$$

where the first operand of XOR $\oplus$ is the structure function of the system when component i is in state 0, and the second is the structure function when component i is in state 1. For example, a logic derivative for (1.2) according to variable x_2 has the following form:

$$\begin{aligned}\frac{\partial\phi(x_1, x_2, x_3)}{\partial x_2} &= \phi(x_1, 0, x_3) \oplus \phi(x_1, 1, x_3) = (x_1 \wedge 0 \vee x_3) \oplus (x_1 \wedge 1 \vee x_3)\\ &= x_3 \oplus (x_1 \vee x_3) = x_1 \wedge \overline{x_3}.\end{aligned} \tag{1.18}$$

From the resulting logic derivation for variable x_2 it is possible to say, that the change of x_2 value will result in change of the value of the Boolean function ϕ if the variable x_1 has value 1 and the variable x_3 has value 0. Logic derivative (1.18) can be graphically represented as shown in Fig. 1.4, where each state vector and its corresponding function value is on the left side of the picture, result of the logic derivative is on the right side of the picture and each node represents the Boolean operation XOR.

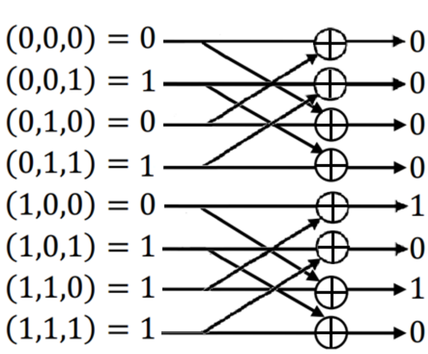


Fig. 1.4 Graphical representation of the logic derivative (1.18)

The logic derivative (1.17) can be used to analyze how a change of component state affects the system state [8], [13]. However, in order to analyze direction of component state change, a direct partial logic derivative (DPLD) is needed. DPLD can be used to analyze how a specific change of component state (from 0 to 1 or from 1 to 0) affects the system functionality (from 0 to 1 or from 1 to 0). This DPLD is defined as follows [26]:

$$\frac{\partial\phi(1\rightarrow 0)}{\partial x_i(1\rightarrow 0)}=\frac{\partial\phi(0\rightarrow 1)}{\partial x_i(0\rightarrow 1)}=\overline{\phi(0_i,\boldsymbol{x})}\wedge\phi(1_i,\boldsymbol{x}), \tag{1.19}$$

where $\wedge$ denotes Boolean operation AND and $\overline{}$ is a negation of the argument interpreted as a Boolean function. There is also another type of partial logic derivative known as inverse partial logic derivative that can be used to analyze how a specific change of component state (from 0 to 1 or from 1 to 0) affects the system functionality (from 1 to 0 or from 0 to 1) and is defined as follows [26]:

$$\frac{\partial\phi(1\rightarrow 0)}{\partial x_i(0\rightarrow 1)}=\frac{\partial\phi(0\rightarrow 1)}{\partial x_i(1\rightarrow 0)}=\overline{\phi(1_i,\boldsymbol{x})}\wedge\phi(0_i,\boldsymbol{x}). \tag{1.20}$$

Given the Boolean function (1.2), the direct and inverse partial logic derivatives with respect to variable x_2 have following forms:

$$\begin{aligned}
\frac{\partial\phi(1\rightarrow 0)}{\partial x_2(1\rightarrow 0)}&=\overline{(x_1\wedge 0\vee x_3)}\wedge(x_1\wedge 1\vee x_3)=\overline{x_3}\wedge(x_1\vee x_3)=x_1\wedge\overline{x_3};\\
\frac{\partial\phi(0\rightarrow 1)}{\partial x_2(0\rightarrow 1)}&=\overline{(x_1\wedge 0\vee x_3)}\wedge(x_1\wedge 1\vee x_3)=\overline{x_3}\wedge(x_1\vee x_3)=x_1\wedge\overline{x_3};\\
\frac{\partial\phi(1\rightarrow 0)}{\partial x_2(0\rightarrow 1)}&=(x_1\wedge 0\vee x_3)\wedge\overline{(x_1\wedge 1\vee x_3)}=x_3\wedge\overline{(x_1\vee x_3)}=0;\\
\frac{\partial\phi(0\rightarrow 1)}{\partial x_2(1\rightarrow 0)}&=(x_1\wedge 0\vee x_3)\wedge\overline{(x_1\wedge 1\vee x_3)}=x_3\wedge\overline{(x_1\vee x_3)}=0.
\end{aligned} \tag{1.21}$$

It is possible to see, that DPLDs are same as the derivation (1.18) and the inverse partial logic derivatives are 0. DPLDs computed in (1.21) can be also graphically represented like in case of logic derivative, which is shown in Fig. 1.5. It is important to point out, that the logic derivative (Fig. 1.4) is composed of direct and inverse partial logic derivatives (Fig. 1.5) that are connected using Boolean operation OR [8] and inverse partial logic derivatives have zero values, because the storage system represented by (1.2) is coherent.

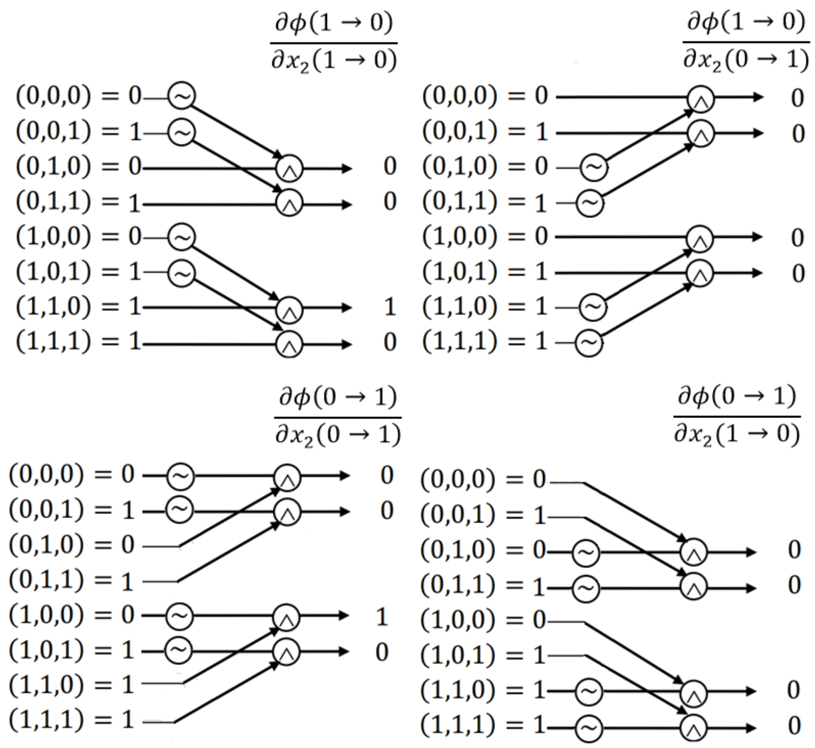


Fig. 1.5 Graphical representation of the DPLDs (1.21)

A DPLD can be computed not just with respect to change of state of one component, but also with respect to simultaneous change of state of two or more components. The latter is known as a DPLD with respect to a vector of changes and has the following form [8]:

$$\frac{\partial\phi(1\to 0)}{\partial(x_i, x_j, \dots)(1,1,\dots)\to(0,0,\dots)} = \overline{\phi(0_i, 0_j, \dots, \boldsymbol{x})} \wedge \phi(1_i, 1_j, \dots, \boldsymbol{x}). \qquad (1.22)$$

This kind of DPLD can be used to analyse not just the same direction of changes of states of components and the system but also the opposite changes or even different changes. For example, for the structure function (1.2) the DPLDs with respect to a vector of changes $(1,1)\to(0,0)$ and $(1,0)\to(0,1)$ of components x_2, x_3 have following forms:

$$\frac{\partial\phi(1\to 0)}{\partial(x_2, x_3)(1,1)\to(0,0)} = \overline{(x_1 \wedge 0 \vee 0)} \wedge (x_1 \wedge 1 \vee 1) = 1 \wedge 1 = 1;$$

$$\frac{\partial\phi(0\to 1)}{\partial(x_2, x_3)(1,1)\to(0,0)} = \overline{(x_1 \wedge 1 \vee 1)} \wedge (x_1 \wedge 0 \vee 0) = 0 \wedge 0 = 0; \qquad (1.23)$$

$$\frac{\partial\phi(1\to 0)}{\partial(x_2, x_3)(0,1)\to(1,0)} = \overline{(x_1 \wedge 1 \vee 0)} \wedge (x_1 \wedge 0 \vee 1) = \overline{x_1} \wedge 1 = \overline{x_1};$$

$$\frac{\partial\phi(0\to 1)}{\partial(x_2,x_3)(0,1)\to(1,0)} = \overline{(x_1 \wedge 0 \vee 1)} \wedge (x_1 \wedge 1 \vee 0) = 0 \wedge x_1 = 0.$$

From DPLDs (1.23), it is possible to conclude that the only interesting change of states of the components x_2, x_3 is $(1,1) \to (0,0)$ but only when the function ϕ changes its state from 1 to 0. All computed DPLDs (1.23) are graphically represented in Fig. 1.6.

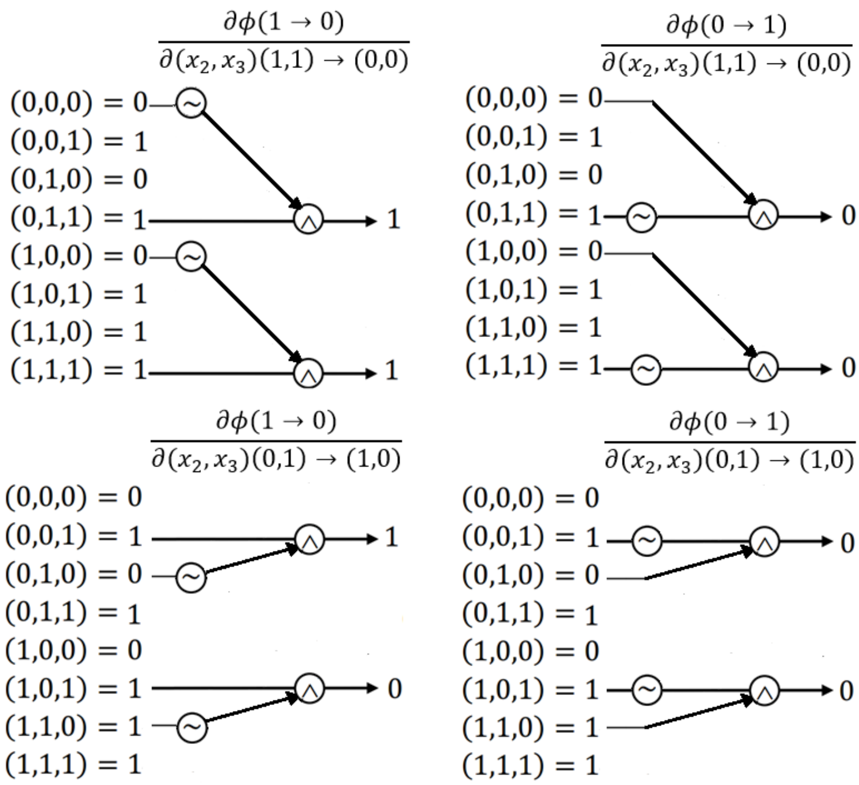


Fig. 1.6 Graphical representation of the DPLDs (1.23)

In reliability analysis, all previously mentioned DPLDs can be mostly used to find critical states of the system [8], [13], which describe situations in which a failure/repair of one or more system components results in a failure/repair of the system. They can also be used to compute importance measures, which will be presented later.

1.3 Survival Signature

Structure function provides an useful and elegant description of the system design, but it has some limitations. For example, in case of system design comparisons, it is more challenging to use this approach, especially with the greater number of system components

[1], [16]. In order to solve this problem, an n-dimensional probability vector known as system signature $\boldsymbol{s}_\tau$ of a coherent system τ with n components can be used [15], [16]. This vector is defined as follows:

$$\mathbf{s}_\tau = (s_1, s_2, \dots, s_n), \tag{1.24}$$

where $s_i = P(T = X_{i:n})$ for $i = 1,2, \dots, n$ represents the probability that the i-th successive component failure at time $X_{i:n}$ results in system failure at time T. It is possible to define s_0, but in coherent systems its value will always be 0, therefore, it will not be taken into account. Because the comparison between designs of topologies of systems with different component characteristics is not a main focus in system signature [16], the Independent and Identically Distributed (IID) assumption for components lifetimes is made.

For example, we will extend the analysis of the data storage system represented by (1.2) as follows: we have to compare the use of RAID 0 (marked as τ_1) or RAID 1 (marked as τ_2) in the upper branch in case of system reliability. In RAID 1, data is written into two drives identically, which means that data can be read from any drive. In addition, RAID 1 will operate successfully as long as at least one drive is functioning [27]. Therefore, the structure function that will represent the second topology with RAID 1 has the following form:

$$\phi_{\tau_2}(x_1, x_2, x_3) = (x_1 \vee x_2) \vee x_3. \tag{1.25}$$

According to the topologies τ_1 and τ_2 represented by (1.2) and (1.25), we can compute the system signature for each topology and then compare them. The failure times of all components of each topology of the data storage system can be ordered in 6 ways and thanks to the IID, they have the same chance to occur. The time $X_{i:n}$ for each permutation of the component failure times can be seen in Tab. 1.1. It follows that the system signature for τ_1 is $\boldsymbol{s}_{\tau_1} = (0,2/3,1/3)$ and for τ_2 is $\boldsymbol{s}_{\tau_2} = (0,0,1)$. It is clear from those system signatures that the topology τ_2 is superior because in this case the system will fail only during the third successive component failure and not mostly during the second successive component failure as in topology τ_1.

Tab. 1.1 The ordered component failure time which causes failure of both system topologies

Ordered Component Failure Times	**Time $X_{i:n}$ in τ_1**	**Time $X_{i:n}$ in τ_2**
$x_1 < x_2 < x_3$	$X_{3:3}$	$X_{3:3}$
$x_1 < x_3 < x_2$	$X_{2:3}$	$X_{3:3}$
$x_2 < x_1 < x_3$	$X_{3:3}$	$X_{3:3}$
$x_2 < x_3 < x_1$	$X_{2:3}$	$X_{3:3}$
$x_3 < x_1 < x_2$	$X_{2:3}$	$X_{3:3}$
$x_3 < x_2 < x_1$	$X_{2:3}$	$X_{3:3}$

System signature $\mathbf{s}_\tau$ can be used for topology comparison, but if the IID probabilities of components to be in working state p or components lifetimes distribution that has cumulative distribution function $Q(t)$ are known, then $\mathbf{s}_\tau$ can be used to compute reliability R or reliability function $R(t)$ as follows [15], [16]:

$$R = \sum_{i=1}^{n} s_i * \sum_{j=n-i+1}^{n} \binom{n}{j} * p^j * q^{n-j}\,; \tag{1.26}$$

$$R(t) = \sum_{i=1}^{n} s_i * \sum_{j=n-i+1}^{n} \binom{n}{j} * \left(1 - F(t)\right)^j * (F(t))^{n-j}. \tag{1.27}$$

It is clear from (1.26) and (1.27) that the lifetime of a coherent system with IID components depends on the system topology only through the system signature.

For example, we will continue with the previous topologies τ_1 and τ_2 of the data storage system. If we want to compute their reliability R and reliability function $R(t)$, we need to know the probability p and lifetime distribution $Q(t)$ of the components. In this example, we will be working with values $p = 0.8$ and components failure times will have exponential distribution with $\lambda = 1/5{,}000$ days. We will firstly compute the reliability of each topology by using previously mentioned p as follows:

$$R_{\tau_1} = 0 * \binom{3}{3} * 0.8^3 * 0.2^0 + \frac{2}{3} * \left(\binom{3}{3} * 0.8^3 * 0.2^0 + \binom{3}{2} * 0.8^2 * 0.2^1\right) + \frac{1}{3} * \left(\binom{3}{3} * 0.8^3 * 0.2^0 + \binom{3}{2} * 0.8^2 * 0.2^1 + \binom{3}{1} * 0.8^1 * 0.2^2\right) = 0.928; \quad (1.28)$$

$$R_{\tau_2} = 0 * \binom{3}{3} * 0.8^3 * 0.2^0 + 0 * \left(\binom{3}{3} * 0.8^3 * 0.2^0 + \binom{3}{2} * 0.8^2 * 0.2^1\right) + 1 * \left(\binom{3}{3} * 0.8^3 * 0.2^0 + \binom{3}{2} * 0.8^2 * 0.2^1 + \binom{3}{1} * 0.8^1 * 0.2^2\right) = 0.992. \quad (1.29)$$

From the calculated reliability R_{τ_1} and R_{τ_2}, it is oblivious that the topology τ_2 is more reliable than the topology τ_1, which corresponds with the results of the comparison of both system signatures. The next step is computation of the reliability function for the both topologies by using (1.27). Time courses of $R_{\tau_1}(t)$ and $R_{\tau_2}(t)$ are depicted in Fig. 1.7, where $R_{\tau_1}(t)$ is shown as a blue dotted line and $R_{\tau_2}(t)$ is shown as a yellow solid line. It is possible to see that the topology τ_2 has always same or better reliability as the days flow and the value of $R_{\tau_2}(t)$ decreasing much slower and smoother that in case of $R_{\tau_1}(t)$.

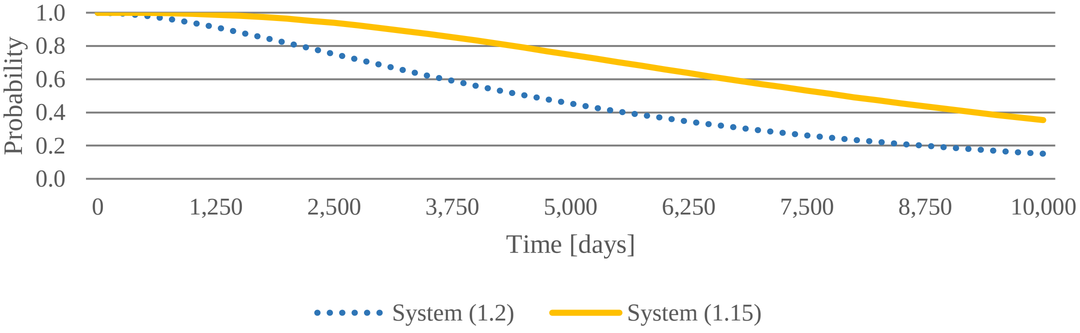


Fig. 1.7 Reliability function of both topologies

The system signature is a useful approach to assist reliability analysis for systems with n components with IID failure times, in which the structure of the system is separated from the random failure times of the components. But this approach has its limitations. The system signature was introduced solely for system with a single type of components with IID failure times. In [15] authors showed the generalization of system signature for systems with several types of components, but deriving this generalization is complex and it does not separate the system structure and the failure time distributions as it was for a single type of components.

Therefore, an alternative to the system signature which can fulfil a similar role known as survival signature was proposed in [15]. Survival signature $\Phi(l)$ for $l = 0,1, \dots, n$ is defined as the probability that the system with n components is working if exactly l system components are in working state [15]. For coherent systems the survival signature has the following form:

$$\Phi(l) = \binom{n}{l}^{-1} * \sum_{x \in S_l} \phi(\boldsymbol{x}), \tag{1.30}$$

where S_l is a set of all state vectors $\boldsymbol{x}$ with exactly l working system components, i.e. $\sum_{i=1}^{n} x_i = l$. In case of $l = 0$ for the coherent systems, the survival signature will always have value 0, and in case of $l = n$, the survival signature will have value 1. As it was with system signature, it is possible to compute the reliability R and the reliability function $R(t)$ using survival signature. If the IID probabilities of components to be in working state p or components lifetimes distribution that has cumulative distribution function $F(t)$ are known, as follows:

$$R = \sum_{l=0}^{n} \Phi(l) * P(C = l) = \sum_{l=0}^{n} \Phi(l) * \binom{n}{l} * p^l * q^{n-l}; \tag{1.31}$$

$$R(t) = \sum_{l=0}^{n} \Phi(l) * P(C_t = l) = \sum_{l=0}^{n} \Phi(l) * \binom{n}{l} * \big(1 - F(t)\big)^l * (F(t))^{n-l}, \tag{1.32}$$

where C and C_t represents the number of components in the system that are in working state or that are in working state at specified time point $t > 0$, i. e. $C, C_t \in \{0,1, \dots, n\}$.

For example, consider the two topologies τ_1 and τ_2 of the data storage system marked as it was in case of computation of the system signature. We can easily compute their survival signatures according to the (1.30) for each $l \in \{0,1,2,3\}$. For example, in case of $l = 1$, there are three state vectors $(1,0,0), (0,1,0)$ and $(0,0,1)$ that represent the situation with exactly one working component. In case of topology τ_1, only for one state vector $(0,0,1)$ is system in working state, which means that $\Phi(1) = \frac{1}{3}$. In case of topology τ_2, the system will be in working state for all state vectors, which means that $\Phi(1) = \frac{3}{3} = 1$. Computed values of survival signatures for both topologies can be seen in Tab. 1.2.

Tab. 1.2 Computed survival signatures for both topologies

Number of working components l	$\Phi(l)$ **of** τ_1	$\Phi(l)$ **of** τ_2
0	0	0
1	1/3	1
2	1	1
3	1	1

In order to compute the reliability R and reliability function $R(t)$ of both topologies, we will use the previously stated values of probability p and lifetime distribution $F(t)$ of the components, i.e. $p = 0.8$ and lifetime distribution will have exponential distribution with $\lambda = 1/5{,}000$ days. The reliability of each topology can be computed using (1.31) as follows:

$$R_{\tau_1} = 0 * \binom{3}{0} * 0.8^0 * 0.2^3 + \frac{1}{3} * \binom{3}{1} * 0.8^1 * 0.2^2 + 1 * \binom{3}{2} * 0.8^2 * 0.2^1 + 1 * \binom{3}{3} * 0.8^3 * 0.2^0 = 0.928; \tag{1.33}$$

$$R_{\tau_2} = 0 * \binom{3}{0} * 0.8^0 * 0.2^3 + 1 * \binom{3}{1} * 0.8^1 * 0.2^2 + 1 * \binom{3}{2} * 0.8^2 * 0.2^1 + 1 * \binom{3}{3} * 0.8^3 * 0.2^0 = 0.992. \tag{1.34}$$

As we can see, the results are same as in case of system signature. The same apply for the computation of the reliability function for the both topologies by using (1.32).

The survival signature (1.30) can be easily generalized to systems with $K \geq 2$ types of components [15]. Survival signature $\Phi(l_1, l_2, \dots, l_K), l_k = 0,1, \dots, n_k$ is defined as the probability that the system with n components is working if exactly l_k system components of type k are in working state for each $k = 1,2, \dots, K$ and has the following form:

$$\Phi(l_1, l_2, \dots, l_K) = \left[\prod_{k=1}^{K} \binom{n_k}{l_k}^{-1}\right] * \sum_{\boldsymbol{x} \in S_{l_1, l_2, \dots, l_K}} \phi(\boldsymbol{x}), \tag{1.35}$$

where $S_{l_1, l_2, \dots, l_K}$ is a set of all state vectors $\boldsymbol{x}$ with exactly $l_1, l_2, \dots, l_K$ working system components. If the IID probabilities of components of type k to be in working state p_k or components lifetimes distribution of type k that has cumulative distribution function $F_k(t)$ for each $k = 1,2, \dots, K$ are known and p_k or $F_k(t)$ of components with different types are independent, then R and $R(t)$ of the system with $K \geq 2$ types of system components can be computed as follows:

$$R = \sum_{l_1=0}^{n_1}\sum_{l_2=0}^{n_2}\cdots\sum_{l_K=0}^{n_K}\Phi(l_1,l_2,\dots,l_K) * P\left(\bigcap_{k=1}^{K}\{C^k = l_k\}\right)$$

$$= \sum_{l_1=0}^{n_1}\sum_{l_2=0}^{n_2}\cdots\sum_{l_K=0}^{n_K}\left[\Phi(l_1,l_2,\dots,l_K)\prod_{k=1}^{K}P(C^k = l_k)\right] \quad (1.36)$$

$$= \sum_{l_1=0}^{n_1}\sum_{l_2=0}^{n_2}\cdots\sum_{l_K=0}^{n_K}\left[\Phi(l_1,l_2,\dots,l_K)\prod_{k=1}^{K}\binom{n_k}{l_k} * (p_k)^{l_k} * (q_k)^{n_k-l_k}\right];$$

$$R(t) = \sum_{l_1=0}^{n_1}\sum_{l_2=0}^{n_2}\cdots\sum_{l_K=0}^{n_K}\left[\Phi(l_1,l_2,\dots,l_K)\prod_{k=1}^{K}P(C_t^k = l_k)\right]$$

$$= \sum_{l_1=0}^{n_1}\sum_{l_2=0}^{n_2}\cdots\sum_{l_K=0}^{n_K}\left[\Phi(l_1,l_2,\dots,l_K)\prod_{k=1}^{K}\binom{n_k}{l_k} * \left(1-F_k(t)\right)^{l_k} * (F(t))^{n_k-l_k}\right], \quad (1.37)$$

where C^k and C_t^k represents the number of components of type k in the system that are in working state or that are in working state at time $t > 0$, i. e. $C^k, C_t^k \in \{0,1,\dots,n_k\}$ for $k = 1,2,\dots,K$. Computation (1.36) and (1.37) may not be straightforward, but they are much easier than in case system signature as is shown in [15] and the information about system structure is fully separated from the information about the failure times of the components.

For illustration, we will use the previous example, but in this case we will use two types of HDDs in data storage system. Components represented by Boolean variables x_1 and x_3 will be of type 1 with $p_1 = 0.8$ and lifetime distribution $F_1(t)$ will have exponential distribution with $\lambda = 1/5{,}000$ days. Component represented by Boolean variable x_2 will be of type 2 with $p_2 = 0.9$ and lifetime distribution $F_2(t)$ will have exponential distribution with $\lambda = 1/8{,}000$ days. Firstly, we will compute the survival signatures according to the (1.30) for each $l_1 \in \{0,1,2\}$ and $l_2 \in \{0,1\}$. For example, in case of $l_1 = 1$ and $l_2 = 0$, there are two state vectors $(1,0,0)$ and $(0,0,1)$ that represent the situation with exactly one working component of type 1 and zero working components of type 2. In case of topology τ_1, the system is in working state only for one state vector $(0,0,1)$, which means that $\Phi(l_1,l_2) = \frac{1}{2}$. In case of topology τ_2, the system will be in working state for all state vectors, which means that $\Phi(l_1, l_2) = \frac{2}{2} = 1$. Computed values of all survival signatures for both topologies can be seen in Tab. 1.3.

Tab. 1.3 Computed survival signatures for both topologies with two types of components

Number of working components l_1	**Number of working components l_2**	$\Phi(l_1, l_2)$ **of** τ_1	$\Phi(l_1, l_2)$ **of** τ_2
0	0	0	0
0	1	0	1
1	0	1/2	1
1	1	1	1
2	0	1	1
2	1	1	1

As a next step we will compute R for each topology using (1.36) and it is shown in (1.38) and (1.39). As we can see, R_{τ_1} is less than R_{τ_2} as it was in previous examples, but both values are greater in contrast to the previous examples, which is caused by more reliable type of HDD added as a component x_2.

$$\begin{aligned} R_{\tau_1} = 0 * \left(\binom{2}{0} * 0.8^0 * 0.2^2 * \binom{1}{0} * 0.8^0 * 0.2^1\right) + 0 \\ * \left(\binom{2}{0} * 0.8^0 * 0.2^2 * \binom{1}{1} * 0.8^1 * 0.2^0\right) + \frac{1}{2} \\ * \left(\binom{2}{1} * 0.8^1 * 0.2^1 * \binom{1}{0} * 0.8^0 * 0.2^1\right) + 1 \\ * \left(\binom{2}{1} * 0.8^1 * 0.2^1 * \binom{1}{1} * 0.8^1 * 0.2^0\right) + 1 * \\ * \left(\binom{2}{2} * 0.8^2 * 0.2^0 * \binom{1}{0} * 0.8^0 * 0.2^1\right) + 1 * \\ * \left(\binom{2}{2} * 0.8^2 * 0.2^0 * \binom{1}{1} * 0.8^1 * 0.2^0\right) = 0.944; \end{aligned} \tag{1.38}$$

$$R_{\tau_2} = 0 * \left(\binom{2}{0} * 0.8^0 * 0.2^2 * \binom{1}{0} * 0.8^0 * 0.2^1\right) + 1 * \left(\binom{2}{0} * 0.8^0 * 0.2^2 * \binom{1}{1} * 0.8^1 * 0.2^0\right) + 1 * \left(\binom{2}{1} * 0.8^1 * 0.2^1 * \binom{1}{0} * 0.8^0 * 0.2^1\right) + 1 * \left(\binom{2}{1} * 0.8^1 * 0.2^1 * \binom{1}{1} * 0.8^1 * 0.2^0\right) + 1 * \left(\binom{2}{2} * 0.8^2 * 0.2^0 * \binom{1}{0} * 0.8^0 * 0.2^1\right) + 1 * \left(\binom{2}{2} * 0.8^2 * 0.2^0 * \binom{1}{1} * 0.8^1 * 0.2^0\right) = 0.996\,. \quad (1.39)$$

Lastly, we will compute the reliability function for the both topologies by using (1.37). Time courses of $R_{\tau_1}(t)$ and $R_{\tau_2}(t)$ are depicted in Fig. 1.8, where $R_{\tau_1}(t)$ is shown as a blue dotted line and $R_{\tau_2}(t)$ is shown as a yellow solid line. As it was in the previous example, the topology τ_2 has always same or better reliability as the days flow and the value of $R_{\tau_2}(t)$ decreasing much slower and smoother that in case of $R_{\tau_1}(t)$. But in this case, adding more reliable HDD resulted in the improvement in the system reliability, especially in the case of $R_{\tau_2}(t)$.

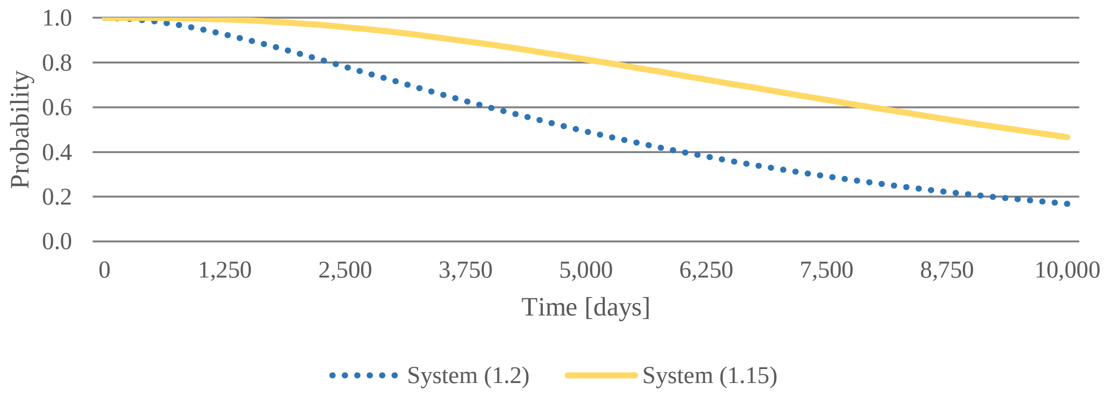


Fig. 1.8 Reliability function of both topologies with two types of components

2 Importance Measures and DPLD

In previous chapter, the theoretical background for reliability analysis like structural function, system signature, survival signature, reliability, unreliability were presented. Reliability and Unreliability are useful measures of the system for reliability analysis, but they do not measure the importance of components for the functioning of the system. Logic differential calculus was also introduced in this chapter as an approach that can be used for structure function to determine, how failure of a system component affects the system operation.

In this chapter, the IMs will be presented as well as standard computation and computation that uses DPLD for structure and reliability IMs. In case of lifetime IMs, they are standardly computed by using the reliability function. The new approach that has been developed for lifetime IMs computation based on the structure function and DPLDs will be shown. The efficiency of new approach will be shown on selected systems in next chapter. This part is based on results presented in [28]. In the second part of this chapter the definition of the DPLDs for the survival signature is considered. Their computation and usability will be demonstrated on the selected systems in next chapter.

2.1 Importance Measures

An important part of reliability analysis is an estimation of influence of a component or a group of components on system operation. Such estimation is implemented by importance measures (IMs) [14] and can be used, for example, to optimize system reliability or to plan its maintenance. There are many IMs, and each of them takes into account different factors that make a system component more important than others. According to [14], IMs can be divided into three categories: structure, reliability, and lifetime IMs.

2.1.1 Structure Importance Measures

These IMs are used to calculate importance of components according to their placement in the system, what means they do not take the reliability of components into account [14]. One of the most commonly known structure IMs is known as Structure

Importance (SI). SI is defined as a relative number of state vectors at which a failure of component i results in system failure. This measure can be computed as [14]:

$$\mathrm{SI}_i = \frac{\sum_{\{(0_i,\boldsymbol{x})\}}(\phi(1_i,\boldsymbol{x}) - \phi(0_i,\boldsymbol{x}))}{2^{n-1}} = \frac{\sum_{\{(1_i,\boldsymbol{x})\}}(\phi(1_i,\boldsymbol{x}) - \phi(0_i,\boldsymbol{x}))}{2^{n-1}}, \tag{2.1}$$

where $\{(0_i,\boldsymbol{x})\}$ represent the set of state vectors and those state vectors have the i-th element 0 and 1 for $\{(1_i,\boldsymbol{x})\}$. Let us consider the data storage system represented by (1.2) as an example for SI calculation. If we want to compute SI for each HDD, we need to find all the state vectors, for which their second value has value 0 and we need to compute $\phi(1_i,\boldsymbol{x}) - \phi(0_i,\boldsymbol{x})$ for them. For HDD 2 that is represented by Boolean variable x_2 is $\mathrm{SI}_2 = \frac{1}{4} = 0.25$, because for one state vector (1,0,0) out of four (0,0,0),(0,0,1),(1,0,0),(1,0,1) has expression $\phi(1_i,\boldsymbol{x}) - \phi(0_i,\boldsymbol{x})$ value 1. As for the other HDDs, $\mathrm{SI}_1 = \frac{1}{4} = 0.25$ and $\mathrm{SI}_3 = \frac{3}{4} = 0.75$. It is possible to see that the most important component according to its placement is HDD 3 with $\mathrm{SI}_3 = 0.75$.

By using logic differential calculus, especially DPLDs, it is possible to compute SI as follows [13]:

$$\mathrm{SI}_i = \mathrm{TD}\left(\frac{\partial\phi(1 \to 0)}{\partial x_i(1 \to 0)}\right), \tag{2.2}$$

where $\mathrm{TD}(.)$ represents the truth density of the argument interpreted as a Boolean function. This value agrees with the relative number of vectors for which the argument takes a nonzero value. Let us consider the data storage system represented by (1.2) as an example for SI calculation. If we want to compute SI for each HDD, we need firstly to compute DPLDs for each HDD by using (1.19) and then compute their truth density. For HDD 2 that is represented by Boolean variable x_2, its DPLD $\frac{\partial\phi(0\to1)}{\partial x_2(0\to1)}$ is $x_1 \wedge \overline{x_3}$, which means that $\mathrm{SI}_2 = \mathrm{TD}(x_1 \wedge \overline{x_3}) = 0.25$. This is due to the fact that only one out of four DPLDs has value 1, and that is when the variable x_1 has value 1, i.e. HDD 1 is working and the variable x_3 has value 0, i.e. HDD 3 is not working. As for the other HDDs, $\mathrm{SI}_1 = 0.25$ and $\mathrm{SI}_3 = 0.75$.

2.1.2 Reliability Importance Measures

Reliability IMs, unlike the Structure IMs, take into account not only system structure in form of structure function but also the probabilities of the components functioning and failure [14]. The most known reliability IM is Birnbaum's Importance (BI), which takes into account system topology and the probabilities of the components functioning and failed. This measure can be computed using reliability as follows [14]:

$$\mathrm{BI}_i = \frac{\partial R}{\partial p_i}, \tag{2.3}$$

and it agrees with the probability that a failure of component i results in system failure, i.e., with the probability that the component is critical for the system. We will show its computation at the data storage system represented by (1.2) with same type of HDD with probability $p = 0.8$ of functioning during mission time. As a first step, we will compute partial derivative for each HDD and then use it to compute BI according to (2.3). For HDD 2 that is represented by Boolean variable x_2, its $\frac{\partial R}{\partial p_2}$ is $p_1 - p_1 * p_3$ and therefore $\mathrm{BI}_2 = 0.8 - 0.8 * 0.8 = 0.16$. As for the other HDDs, $\mathrm{BI}_1 = p_2 - p_2 * p_3 = 0.16$ and $\mathrm{BI}_3 = 1 - p_1 * p_2 = 0.36$. From those values we can conclude, that a failure of HDD 3 is the most problematic, because this failure will result in system failure with the highest probability.

Alternatively, BI can be also computed using DPLD as follows [13]:

$$\mathrm{BI}_i = \mathrm{Pr}\left\{\frac{\partial\phi(1 \to 0)}{\partial x_i(1 \to 0)} = 1\right\}. \tag{2.4}$$

We will show its computation at the data storage system represented by (1.2) with same type of HDD with probability $p = 0.8$ of functioning during mission time. As it was in case of SI, we will firstly compute DPLD (1.19) for each HDD and then use it to compute BI according to (2.4). For HDD 2 that is represented by Boolean variable x_2, its DPLD $\frac{\partial\phi(0\to1)}{\partial x_2(0\to1)}$ is $x_1 \wedge \overline{x_3}$ and by transforming it into a probabilistic form we get $\mathrm{BI}_2 = p_1 - p_1 * p_3 = 0.16$. As for the other HDDs, $\mathrm{BI}_1 = p_2 - p_2 * p_3 = 0.16$ and $\mathrm{BI}_3 = 1 - p_1 * p_2 = 0.36$.

Another useful type of reliability IM is Criticality Importance (CI). This IM extends the BI and it corresponds to the probability that system failure has been caused by a failure of component i given that the system has failed [14]. This is shown by following formula:

$$\mathrm{CI}_i = \mathrm{BI}_i \frac{q_i}{F}, \tag{2.5}$$

and it can be used to find components whose failures have resulted in system failure with the greatest probability when we know that the system has failed. It is typically used in system maintenance to identify components whose repairs will result in system repair with the greatest probability [3], [14]. As it was for BI, we will compute the CI for each HDD in the data storage system represented by (1.2). In case of HDD 1, by using its $\mathrm{BI}_1 = 0.16$, the probability of HDD failure during mission time $q = 1 - p = 0.2$ and system unreliability $F = 0.072$, we can then compute $\mathrm{CI}_1 = 0.16 * \frac{0.2}{0.072} = 0.444$. As for the other HDDs, $\mathrm{CI}_2 = 0.444$ and $\mathrm{CI}_3 = 1$, which means that if we choose to repair HDD 3 if it is in non-working state, the system will start to work given that the system was failed.

BI and CI are very useful in importance analysis of a single system component and a lot of other measures are derived from them [13], [14]. However, it is not usable for importance analysis of a pair of the components. In order to perform importance analysis that provides additional information, which the IMs of a single component cannot convey, IMs of pairs of system components are used. One such IM is the joint reliability importance (JRI), which can be computed for two components i and j in the following manner [14], [29]:

$$\mathrm{JRI}_{i,j} = \frac{\partial^2 R}{\partial p_i \partial p_j}. \tag{2.6}$$

The JRI is useful for analyzing the interactions of two components with each other when their reliabilities change. This IM may produce negative or positive values, where sign represents the type of the interaction, and the absolute value quantifies the degree of the interaction between the two components with respect to reliability of the system [14], [29]. If $\mathrm{JRI}_{i,j} < 0$, then component i (j) is more important with respect to the system reliability when component j (i) is failed than when component j (i) is functioning. However, if $\mathrm{JRI}_{i,j} > 0$, then component i (j) is more important for system operation when component j (i) is functioning than when component j (i) fails. As an example, we will continue with data storage system with reliability (1.12) and we will compute JRI for each possible pair of HDDs. In case of HDDs 1 and 2, $\mathrm{JRI}_{1,2} = \frac{\partial^2 R}{\partial p_1 \partial p_2} = 1 - p_3 = 0.2$, which means that HDD 1 is more important for the system reliability if HDD 2 is functioning. This agrees with their serial placement in the given topology of the system. On the other hand, $\mathrm{JRI}_{1,3} = \frac{\partial^2 R}{\partial p_1 \partial p_3} = -p_2 = -0.8$ and $\mathrm{JRI}_{2,3} = \frac{\partial^2 R}{\partial p_2 \partial p_3} = -p_1 = -0.8$ are both negative, which means that HDD 1 and 2 are more important for the system reliability if HDD 3 is failed. This corresponds to their placement in different branches in parallel topology of the analysed system.

The previously mentioned IMs can be used to find the most important components according to the selected criterion or to see how two components interact with each other when their reliability changes. However, if there is a need to analyse how a simultaneous failure of two system components can affect the system, these measures cannot be used. For this purpose, we developed a new IM [30]. This new IM is based on DPLD (1.22) that can find situations, in which a simultaneous failure of two components results in system failure. In order to evaluate importance of two components with respect to their simultaneous failure, the probability of these situations must be computed. Therefore, the new importance measure, which can be named as a Failure Importance (FI), is computed as follows:

$$\mathrm{FI}_{i,j} = \Pr\left\{\frac{\partial\phi(1\to 0)}{\partial(x_i,x_j)((1,1)\to(0,0))} = 1\right\}. \tag{2.7}$$

This measure helps to understand how a simultaneous failure of system components i and j influences system operation. As an example, we will again use the data storage system represented by (1.2) for computation of FI for each combination of two HDDs. For HDDs 1 and 2, their DPLD $\frac{\partial\phi(1\to 0)}{\partial(x_1,x_2)((1,1)\to(0,0))} = \overline{x_3}$, therefore, their $\mathrm{FI}_{1,2} = 1 - p_3 = 0.2$, which shows that the simultaneous failure of the HDD 1 and 2 slightly affect the system functionality. On the other hand, for combinations HDD 1 with HDD 3 and HDD 2 with HDD 3, $\mathrm{FI}_{1,3} = \mathrm{FI}_{2,3} = 1$, which means that the system will surely fail with simultaneous failure of those HDDs.

2.1.3 Lifetime Importance Measures

Reliability IMs assume that the state probabilities p_i and q_i of the components are known, and they do not depend on time. If we know how these probabilities changes in time, we can investigate how BI and CI of the components vary during system mission. For this purpose, lifetime IMs are used [14]. These IMs depend on the positions of components within the system and the components lifetime distributions.

First IM, that will be presented in this chapter is time-dependent BI for component i at time t. This IM agrees with the probability that the system is in a state at time t in which component i is critical for the system, and it is standardly computed by partial differentiation of reliability function $R(t)$ according to $P_i(t)$ [14]:

$$\mathrm{BI}_i(t) = \frac{\partial R(t)}{\partial P_i(t)}. \tag{2.8}$$

We will show its computation at the data storage system represented by (1.2) with same type of HDD that are independent and they have exponential distribution with $\lambda = 1/5{,}000$ days as lifetime distribution. By using this system reliability function (1.15) it is possible to compute derivation for each HDD and then use it to compute time-dependent BI according to the (2.8). Therefore, their time-dependent BI are $\mathrm{BI}_1(t) = \frac{\partial R(t)}{\partial P_1(t)} = P_2(t) - P_2(t) * P_3(t)$, $\mathrm{BI}_2(t) = \frac{\partial R(t)}{\partial P_2(t)} = P_1(t) - P_1(t) * P_3(t)$ and $\mathrm{BI}_3(t) = \frac{\partial R(t)}{\partial P_3(t)} = 1 - P_1(t) * P_2(t)$. Time courses of those IMs are depicted in Fig. 2.1, where $\mathrm{BI}_1(t)$ is represented by green dash line, $\mathrm{BI}_2(t)$ is represented by red dotted line and $\mathrm{BI}_3(t)$ is represented by blue solid line. From those time courses we can conclude that a failure of HDD 3 is the most problematic throughout the whole time, because this failure will result in system failure with the highest probability and its value raises sharply for 5,000 days. As for the other HDDs, their importance raises slightly for around 3,000 days and then it started slowly decreased. This is mostly caused by their placement in the series and by the fact, that the HDD 3 is with them in the parallel topology.

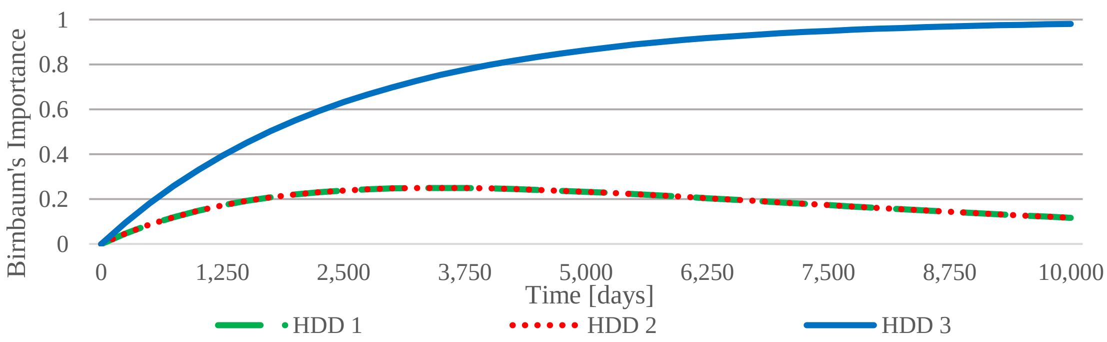


Fig. 2.1 Time-dependent BI measures for HDDs of the storage system

The previously mentioned approach for computation of the time-dependent BI uses reliability function that can be obtained by using transformation of the structure function. This process is shown in Fig. 2.2 on left side. Another possibility, that we suggest in this work, is to use DPLD for computation of the time-dependent BI. This approach will allow us to use other IMs that can be computed using DPLD in time-dependent reliability analysis as is shown in Fig. 2.2 on the right side and it will be defined as follows:

$$\mathrm{BI}_i(t) = \Pr\left\{\frac{\partial Z(1 \to 0, t)}{\partial x_i(1 \to 0, t)} = 1\right\}. \tag{2.9}$$

This new way for computation of the time-dependent BI is based on the same approach as was shown in section 1.1 in case of the structure function, thanks to the fact, that the structure function and the DPLD are both Boolean function and in case of DPLD, the meaning of the reliability (availability) function will change to time dependent BI. We will show its computation on the data storage system represented by (1.2). According to this approach, we need to compute DPLDs for each HDD. These DPLDs are shown in Tab. 2.1. In the next step, we transform them into time-dependent probability form and therefore we obtain the time-dependent BI for each HDD. These BIs are $\mathrm{BI}_1(t) = P_2(t) * Q_3(t) = P_2(t) - P_2(t) * P_3(t)$, $\mathrm{BI}_2(t) = P_1(t) * Q_3(t) = P_1(t) - P_1(t) * P_3(t)$ and $\mathrm{BI}_3(t) = Q_1(t) + Q_2(t) - Q_1(t) * Q_2(t) = 1 - P_1(t) * P_2(t)$. As we can see, those time-dependent BIs are exactly same as in case of normally used approach.

Tab. 2.1 DPLD for each HDD of the storage system

Number of HDD	**DPLD**
1	$x_2 \wedge \overline{x_3}$
2	$x_1 \wedge \overline{x_3}$
3	$\overline{x_2} \vee \overline{x_2}$

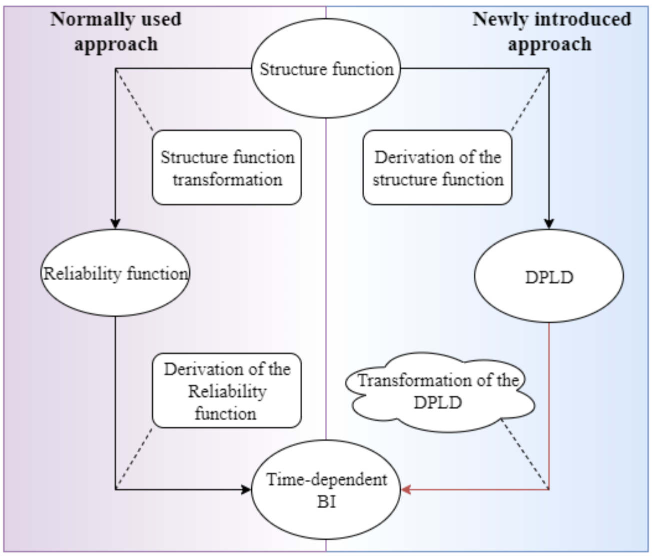


Fig. 2.2 Showing the new approach for computation of the time-dependent BI

Another useful time-dependent IM is $\mathrm{CI}_i(t)$. This IM can be computed from $\mathrm{BI}_i(t)$ as follows [14]:

$$\mathrm{CI}_i(t) = \mathrm{BI}_i(t)\frac{Q_i(t)}{F(t)}, \tag{2.10}$$

and it corresponds to the probability that component i has failed by time t and that component i is critical for the system at time t, given that the system has failed by time t [14]. We will show its computation on the data storage system represented by (1.2). By using the previous computed time-dependent BI and by using (2.10), the time-dependent CI for each HDD are $\mathrm{CI}_1(t) = \mathrm{BI}_1(t)\frac{Q_1(t)}{F(t)} = \frac{Q_1(t)*Q_3(t)-Q_1(t)*Q_2(t)*Q_3(t)}{Q_1(t)*Q_3(t)+Q_2(t)*Q_3(t)-Q_1(t)*Q_2(t)*Q_3(t)}$, $\mathrm{CI}_2(t) = \mathrm{BI}_2(t)\frac{Q_2(t)}{F(t)} = \frac{Q_2(t)*Q_3(t)-Q_1(t)*Q_2(t)*Q_3(t)}{Q_1(t)*Q_3(t)+Q_2(t)*Q_3(t)-Q_1(t)*Q_2(t)*Q_3(t)}$ and $\mathrm{CI}_3(t) = \mathrm{BI}_3(t)\frac{Q_3(t)}{F(t)} = \frac{Q_1(t)*Q_3(t)+Q_2(t)*Q_3(t)-Q_1(t)*Q_2(t)*Q_3(t)}{Q_1(t)*Q_3(t)+Q_2(t)*Q_3(t)-Q_1(t)*Q_2(t)*Q_3(t)} = 1$. Their time courses can be seen in Fig. 2.3, where $\mathrm{CI}_1(t)$ is represented by green dash line, $\mathrm{CI}_2(t)$ is represented by red dotted line and $\mathrm{CI}_3(t)$ is represented by blue solid line. From those time courses, we can conclude that repair of the HDD 3 will surely results in system repair in any time point if we know that the system has failed because its value is always 1. This is caused by its placement in one branch in the parallel topology of the system. As for the other HDDs, their value of time-dependent CI slowly decreases as time flows, which means that repair of one of those HDDs will result in system repair with less probability at a later time.

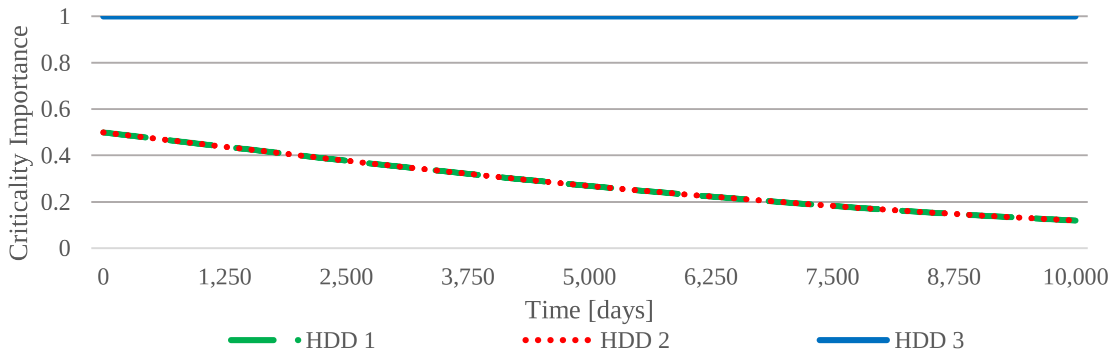


Fig. 2.3 Time-dependent CI measures for HDDs of the storage system

As in the case of other reliability IMs mentioned in the previous chapter, JRI and FI can be defined as time-independent (if state probabilities p_i or q_i of the components are known) or time-dependent (if functions $P_i(t)$ or $Q_i(t)$ defining behavior of the components over time are known).

2.2 Logic Differential Calculus in Survival Signature

In previous section we showed how the DPLD can be used for computation of the lifetime IMs such as BI and CI. As we presented, the IMs based on DPLDs allow computing all types of IMs: structure, reliability and lifetime, logic differential calculus can be viewed as a mathematical methodology unifying these three core concepts of importance analysis into one complex framework. However, a main problem behind the approach presented above is a fact that the number of possible state vectors in structure function can be really big in case of systems composed of huge amount of components, therefore, some analysis presented above can be really time-consuming. This implies it is important to develop new approaches that allow represent the system under investigation in a more compact way. One such approach is System Signature. If we want to use the mathematical apparatus presented above in reliability analysis based on System Signature we have to expand the concept of DPLDs and its applications on such a representation. This issue is addressed in the rest of the chapter.

DPLD for Boolean function with respect to variable x_i according to (1.19) allows us to indicate state vector $(x_1, x_2, \dots, x_n)$ of the function for which the specified change of the variable x_i results the specified change of the function. In terms of reliability analysis this derivative allows indicating critical states of system that agree with state vectors of components for which the breakdown of the i-th component cause the failure of the system. In case of the survival signature (1.35), the DPLD can have interesting interpretations. In this part, the new possible derivatives are considered, which are focused on the case of the component breakdown and system degradation.

The first DPLD for survival signature indicates the possibility of the system failure for fixed number of working components of the specified type if one of components of this type breakdowns:

$$\frac{\partial \Phi(l_1, \dots, l_K) \downarrow}{\partial l_k(a \to a-1)} = \begin{cases} 1, & \Phi(l_1, \dots, a_k, \dots, l_K) > \Phi(l_1, \dots, a_k - 1, \dots, l_K) \\ 0, & \text{otherwise} \end{cases} \tag{2.11}$$

where $a \in \{1,2, \dots, n_k\}$ is a number of working components of the type $k \in \{1,2, \dots, K\}$. This derivative is defined only for $(l_1, \dots, a_k, \dots, l_K)$ working components of each type and its value is non-zero if $\Phi(l_1, \dots, a_k, \dots, l_K) > \Phi(l_1, \dots, a_k - 1, \dots, l_K)$ and otherwise its value is zero. This derivation is based on the integrated direct partial logic derivatives of type 2 that are shown in [31]. This comes from the fact, that the survival signature can be seen as a multi-valued mathematical representation of the analysed system. It is also possible to compute $\mathrm{SI}_{k,a}^{\downarrow}$ for such system by taking definition of the $\mathrm{SI}_{i,s}^{\downarrow}$ in [31] into account as follows:

$$\mathrm{SI}_{k,a}^{\downarrow} = \mathrm{TD}\left(\frac{\partial \Phi(l_1, \dots, l_K) \downarrow}{\partial l_k(a \to a-1)}\right), \tag{2.12}$$

where $\mathrm{TD}(.)$ represents the truth density of the argument interpreted as a Boolean function. $\mathrm{SI}_{k,a}^{\downarrow}$ represents a relative number of situations in which a number of working components a of type k is critical for the system degradation.

We will show how this DPLD and $\mathrm{SI}_{k,a}^{\downarrow}$ are computed on the example of data storage system represented by (1.2) with two types of components that was first described in section 1.3 for survival signature with K types of components. In this system, there are two components 1 and 3 of type 1 and one component 2 of type 2. This means that it is possible to compute first DPLD for change from 2 working components to one and from 1 working components to zero for l_1 and for change from 1 working components to zero for l_2. According to the (2.11), we can conclude that the value of DPLD $\frac{\partial \Phi(l_1,l_2)\downarrow}{\partial l_1(2\to 1)}$ has value 1 in situation, where only two components of type 1 are working because $(\Phi(2,0) = 1) > (\Phi(1,0) = 0.5)$ and has value 0 in situation when all system components are working because $(\Phi(2,1) = 1) \ngtr (\Phi(1,1) = 1)$. As for the $\mathrm{SI}_{k,a}^{\downarrow}$ computation, by using (2.12) and computed DPLDs their values are $\mathrm{SI}_{1,2}^{\downarrow} = \mathrm{TD}\left(\frac{\partial \Phi(l_1,l_2)\downarrow}{\partial l_1(2\to 1)}\right) = 0.5$, $\mathrm{SI}_{1,1}^{\downarrow} = \mathrm{TD}\left(\frac{\partial \Phi(l_1,l_2)\downarrow}{\partial l_1(1\to 0)}\right) = 1$ and $\mathrm{SI}_{2,1}^{\downarrow} = \mathrm{TD}\left(\frac{\partial \Phi(l_1,l_2)\downarrow}{\partial l_2(1\to 0)}\right) = 0.333$. Values of all DPLDs and $\mathrm{SI}_{k,a}^{\downarrow}$ can be seen in Tab. 2.2. It is possible to see that the most crucial change is change of type 1 from one working type to zero because it will always result in decrease of the system signature's value. This can be proved by $\mathrm{SI}_{1,1}^{\downarrow} = 1$. On the other hand, the change of type 2 from one working type to zero is the least

Tab. 2.2 First DPLD for data storage system

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1, l_2)$	$\frac{\partial \Phi(l_1,l_2)\downarrow}{\partial l_1(2\to 1)}$	$\frac{\partial \Phi(l_1,l_2)\downarrow}{\partial l_1(1\to 0)}$	$\frac{\partial \Phi(l_1,l_2)\downarrow}{\partial l_2(1\to 0)}$
0	0	0	-	-	-
0	1	0	-	-	0
1	0	0.5	-	1	-
1	1	1	-	1	1
2	0	1	1	-	-
2	1	1	0	-	0
$\mathrm{SI}_{k,a}^{\downarrow}$			0.5	1	0.333

crucial change because it is significant only in one of the three possible situations, which can be verified by $\mathrm{SI}_{2,1}^{\downarrow}$ value 0.333.

The second DPLD for survival signature indicates the possibility of the system failure if one of the system components of specified type breakdowns:

$$\frac{\partial\Phi(l_1,\dots,l_K)\downarrow}{\partial l_k\downarrow}=\begin{cases}1, & \Phi(l_1,\dots,l_k,\dots,l_K)>\Phi(l_1,\dots,\widetilde{l_k},\dots,l_K)\\ 0, & \text{otherwise}\end{cases} \tag{2.13}$$

or

$$\frac{\partial\Phi(l_1,\dots,l_K)\downarrow}{\partial l_k\downarrow}=\bigcup_{a=1}^{n_k}\frac{\partial\Phi(l_1,\dots,l_K)\downarrow}{\partial l_k(a\rightarrow a-1)} \tag{2.14}$$

where $l_k\in\{1,2,\dots,n_k\}$, and $\widetilde{l_k}=l_k-1$. From (2.14) it is possible to see, that this DPLD can be obtained by conjunction of (2.11) for each $a=1,2,\dots,n_k$. This DPLD can be used to compute $\mathrm{SI}_k^{\downarrow}$ as follows:

$$\mathrm{SI}_k^{\downarrow}=\mathrm{TD}\left(\frac{\partial\Phi(l_1,\dots,l_K)\downarrow}{\partial l_k\downarrow}\right)=\frac{\sum_{a=1}^{n_k}\mathrm{SI}_{k,a}^{\downarrow}}{n_k}, \tag{2.15}$$

and this definition corresponds with the computation of the $\mathrm{SI}_i^{\downarrow}$ defined in [31]. $\mathrm{SI}_k^{\downarrow}$ represents a relative number of situations in which decrease in the number of working components of type k results in system degradation.

We will show how this DPLD and SI is computed by using the same example as we used for the first DPLD. By using (2.13), the DPLD $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}$ for $l_1=1, l_2=0$ has value 1 because $(\Phi(1,0)=0.5)>(\Phi(0,0)=0)$ and the DPLD $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2\downarrow}$ for $l_1=0, l_2=1$ has value 0 because $(\Phi(0,1)=0)\ngtr(\Phi(0,0)=0)$. As for the $\mathrm{SI}_k^{\downarrow}$ computation, by using (2.15) and computed DPLDs their values are $\mathrm{SI}_1^{\downarrow}=\mathrm{TD}\left(\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}\right)=0.75=\frac{0.5+1}{2}$ and $\mathrm{SI}_2^{\downarrow}=\mathrm{TD}\left(\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2\downarrow}\right)=0.333=\frac{0.333}{1}$. All values of the second DPLD and $\mathrm{SI}_k^{\downarrow}$ can be seen in Tab. 2.3. From those values it is possible to see that the most crucial type is type 1 because in only one situation out of four is the value of the second DPLD 0 and as for the second type, it has only one situation out of three, in which the system signature's value will degrade with failure of component with such a type.

Tab. 2.3 Second DPLD for data storage system

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1,l_2)$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2\downarrow}$
0	0	0	-	-
0	1	0	-	0
1	0	0.5	1	-
1	1	1	1	1
2	0	1	1	-
2	1	1	0	0
$\mathrm{SI}_k^{\downarrow}$			0.75	0.333

The third and final DPLD for survival signature shows measure of the system failure if the one of the components of specified type breakdowns:

$$\frac{\partial\Phi(l_1,\dots,l_K)\Downarrow}{\partial l_k\downarrow}=\begin{cases}\xi, & \Phi(l_1,\dots,l_k,\dots,l_K)>\Phi\left(l_1,\dots,\widetilde{l_k},\dots,l_K\right)\\ 0, & \text{otherwise}\end{cases} \tag{2.16}$$

where $\xi=\Phi(l_1,\dots,l_k,\dots,l_K)-\Phi\left(l_1,\dots,\widetilde{l_k},\dots,l_K\right)$ for l_k = 1, 2, .., n_k, $l_k>\widetilde{l_k}$ and $\widetilde{l_k}=l_k-1$. Another formulation of this DPLD is as follows:

$$\frac{\partial\Phi(l_1,\dots,l_K)\Downarrow}{\partial l_k\downarrow}=(n_k)^{-1}\cdot\sum_{x_i\in N_k}\Phi\left(\frac{\partial\phi(1\to0)}{\partial x_i(1\to0)}\right) \tag{2.17}$$

where $\Phi\left(\frac{\partial\phi(1\to0)}{\partial x_i(1\to0)}\right)$ is transformation of each DPLD $\frac{\partial\phi(1\to0)}{\partial x_i(1\to0)}$ based on the rules of the survival signature (1.35) and N_k is a set of all components of type k. The SI for this derivation represents an average degradation in the value of the survival signature by taking into account a decrease in the number of functioning components of type k. This can be represented as follows:

$$\mathrm{SI}_k^{\Downarrow}=\frac{\sum_{\boldsymbol{l}\in S_k}\frac{\partial\Phi(\boldsymbol{l})\Downarrow}{\partial l_k\downarrow}}{n_k*\prod_{i\in M_k}(n_i+1)}, \tag{2.18}$$

where $\boldsymbol{l}=(l_1,\dots,l_K)$ represents a vector of variables that represent number of working components of each type, S_k is a set of all vectors $\boldsymbol{l}$ in which $l_k\in\{1,2,..,n_k\}$ and $l_i\in\{0,1,2,..,n_i\}$ for $i=1,\dots,k-1,k+1,\dots,K$ and M_k is a set $\{1,\dots,k-1,k+1,\dots,K\}$.

We will show how this DPLD and $\mathrm{SI}_k^{\Downarrow}$ are computed by using the same example as for previous DPLDs. By using (2.16), the DPLD $\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_1\downarrow}$ for $l_1=1,l_2=0$ has value $(\Phi(1{,}0)=0.5)-(\Phi(0{,}0)=0)=0.5$ because $(\Phi(1{,}0)=0.5)>(\Phi(0{,}0)=0)$ and the DPLD $\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_2\downarrow}$ for $l_1=0,l_2=1$ has value 0 because $(\Phi(0{,}1)=0)\not>(\Phi(0{,}0)=0)$. In case of $\mathrm{SI}_k^{\Downarrow}$, by using (2.18) their values for each type are $\mathrm{SI}_1^{\Downarrow} = \frac{(0.5+1+0.5+0)}{2*2} = 0.5$ and $\mathrm{SI}_2^{\Downarrow} = \frac{(0+0.5+0)}{1*3} = 0.167$. All values of the third DPLD and $\mathrm{SI}_k^{\Downarrow}$ can be seen in Tab. 2.4. From those values it is possible to see, that the most crucial type is type 1 as it was in case of second DPLD, which is further proved by the $\mathrm{SI}_1^{\Downarrow}$. However, from the third DPLD we can clearly see, that the most critical situation is when one component of the first type fails for $l_1 = 1, l_2 = 1$, because system will surely fail (value of the $\frac{\partial\Phi(1,1)\Downarrow}{\partial l_1\downarrow}$ is 1) and other non-zero values of the third DPLD for type 1 has value 0.5.

Tab. 2.4 Third DPLD for data storage system

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1, l_2)$	$\frac{\partial\Phi(l_1, l_2)\Downarrow}{\partial l_1\downarrow}$	$\frac{\partial\Phi(l_1, l_2)\Downarrow}{\partial l_2\downarrow}$
0	0	0	-	-
0	1	0	-	0
1	0	0.5	0.5	-
1	1	1	1	0.5
2	0	1	0.5	-
2	1	1	0	0
$\mathrm{SI}_k^{\Downarrow}$			0.5	0.167

3 Case Studies

In the previous chapter, the IMs were introduced and described. For structure and reliability IMs, the alternate computation using the logic differential calculus was shown. As for the time-dependent IMs, the new approach for computation of the BI using logic differential calculus was introduced as an continuation from [13], [14]. The FI was also introduced as a new IM, that can be used to investigate how simultaneous failure of two system components can affect the system performance. In the first section in this chapter, the new approach of time-dependent IMs computation based on DPLDs will be presented alongside other described IMs on reliability analysis of three different systems.

In second part in the previous chapter, the new approach for computation of the DPLDs for survival signature was introduced. By using this approach, it is possible to define IMs that do not investigate how a specific component of the system affects the system performance, but how the type of component affect the system performance. This was shown for the SI for each DPLD. In the second section in this chapter, the usage of logic differential calculus for reliability analysis based on survival signature will be shown on four selected systems.

3.1 Case Studies for Lifetime Importance Measures

In this section we will show how the new approach of time-dependent IMs computation based on DPLDs can be used in reliability analysis of three different systems, namely storage system, in which all presented reliability and lifetime IMs are computed, drone fleet, in which we focus on reliability function for homogenous and heterogeneous drone fleet and time-dependent BIs and Cis for each drone in heterogeneous fleet, and surveillance system, in which we compute reliability function and time-dependent BIs and CIs. It is needed to point out that the results from new approach and the result from the standard approach are the same, which was showed in 2.1.3.

3.1.1 Case Study for Storage System

Firstly, we will perform the reliability evaluation of storage system (Fig. 3.1) by the calculation of reliability function of a system based on its structure function and JRI, BI, FI and CI measures with the use of DPLDs.

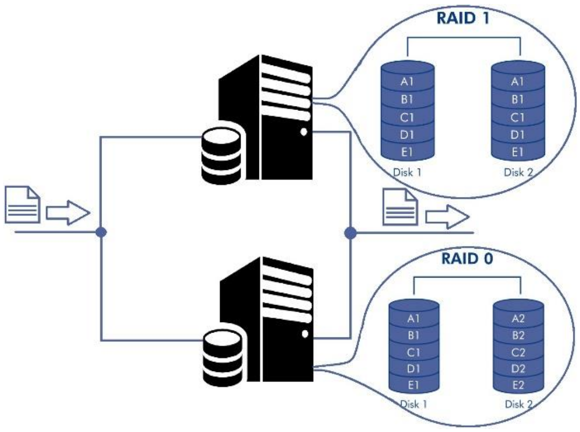


Fig. 3.1 Analysed storage system

The analyzed storage system is composed of two storage units parallel connected in communication network. These units have two HDDs organized in a specific structure called RAID. The first unit, which is in the upper path, consists of two HDDs in RAID 1 and the second unit, which is in the lower path, consists of two HDDs organized in configuration RAID 0. Each of both units has capacity 6 TB, and they are parallel connected to the network and used to store the same data. The HDDs WDC WD30EZRX and HGST HDS5C3030ALA are used in RAID 0, because they have lower read and write speeds than HDDs in RAID 1 and RAID 0 allows us to increase read and write speeds. Thanks to this the storage system is functioning (data can be stored to it or load from it) if at least one unit is working.

In reliability analysis of the storage system, we will focus mostly on storage units, specifically their HDDs, and we will not consider network reliability. According to the description of the storage system, the system can be in one of two possible states: state 0 - agrees with a situation in which data cannot be stored or retrieved; state 1 - it is possible to store data or retrieve them. The components are HDDs, and they can be in one of two states: state 1 represents functioning HDD; state 0 represents HDD failure.

System topology expressed in the form of RBD can be seen in Fig. 3.2. Block x_1 denotes a variable defining state of HDD SEAGATE ST6000DX000 in RAID 1, x_2 is a state variable for HDD WDC WD60EFRX in RAID 1, x_3 represents a variable defining state of HDD SEAGATE ST3000DM001 in RAID 0, and x_4 agrees with state of HDD HGST HDS5C3030ALA in RAID 0. The reliability block diagram, constructed based on the previously introduced description of the storage system, allows us to obtain the structure function of the system:

$$\phi(x_1, x_2, x_3, x_4) = (x_1 \vee x_2) \vee x_3 \wedge x_4. \tag{3.1}$$

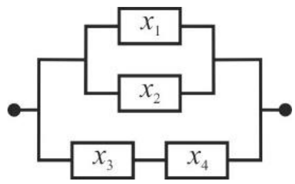


Fig. 3.2 Reliability block diagram of the storage system

If we know the probabilities that the system components are working, i.e., p_i for $i = 1,2,3,4$, we can transform this function into probabilistic function [14], which allows us to compute the probability that the system is functioning:

$$\begin{aligned}(x_1 \vee x_2) \vee x_3 \wedge x_4 \rightarrow (p_1 + p_2 - p_1p_2) + p_3p_4 - \ (p_1 + p_2 - p_1p_2)p_3p_4 \\ = \ p_1 + p_2 - p_1p_2 + p_3p_4 - p_1p_3p_4 - p_2p_3p_4 + p_1p_2p_3p_4.\end{aligned} \tag{3.2}$$

This formula can be used as a base for finding reliability function $R(t)$ of the system. For this purpose, we simply replace probabilities p_i by functions $P_i(t)$, for $i = 1,2,3,4$. Function $P_i(t)$ defines the probability that component i will operate correctly throughout interval $\langle 0, t\rangle$ given that it worked at time 0. Reliability function $P_i(t)$ of component i can be obtained from lifetime distribution $Q_i(t)$ using relationship $P_i(t) = 1 - Q_i(t)$. So we can write:

$$\begin{aligned}R(t) = (1 - Q_1(t)) + (1 - Q_2(t)) - (1 - Q_1(t))(1 - Q_2(t)) \\ + (1 - Q_3(t))(1 - Q_4(t)) \\ - (1 - Q_1(t))(1 - Q_3(t))(1 - Q_4(t)) \\ - (1 - Q_2(t))(1 - Q_3(t))(1 - Q_4(t)) \\ + (1 - Q_1(t))(1 - Q_2(t))(1 - Q_3(t))(1 - Q_4(t)).\end{aligned} \tag{3.3}$$

In this example we will assume that lifetime distributions $Q_i(t)$ of the HDDs agree with exponential distribution [3] with parameter $\lambda = 1/\mathrm{MTTF}$, where MTTF denotes mean time to failure. We obtained MTTF in days for each HDD from data published by Backblaze

Storage company in 2016 [32]. The company publishes quarterly statistics about HDDs that they use in their storage solutions. Based on them, the company estimates Annualized Failure Rate (AFR) for individual models of HDDs. AFR is an estimation of the probability that a device (HDD) will fail during a full year of use [33], and its relation to MTTF in days is as follows [34]:

$$\mathrm{MTTF} = \frac{-365.25}{\ln(1-\mathrm{AFR})}. \tag{3.4}$$

AFRs of the HDDs estimated by the company based on data from April 2013 to December 2016 are shown in Tab. 3.1. MTTFs in days, which we need to obtain lifetime distributions $Q_i(t)$ of the HDDs, were obtained from AFRs by transforming (3.4) into the following form:

$$\mathrm{AFR} = 1 - e^{\frac{-365.25}{\mathrm{MTTF}}}. \tag{3.5}$$

Based on MTTFs presented in Tab. 3.1, we can obtain the reliability function of the storage system. Its time course is presented by the blue dashed curve in Fig. 3.3. In this graph, we can see how reliability of the storage system with increasing time gradually degrades. Using the reliability function, we can compute, for example, that after 1,572 days, the reliability of the system decreases below 0.99.

Tab. 3.1 Properties of HDDs used in the Storage System

Component	**Component data**			
	HDD name	***Capacity [TB]***	***AFR***	***MTTF [days]***
1	SEAGATE ST6000DX000	6	0.0143	25,359
2	WDC WD60EFRX	6	0.0568	6,246
3	SEAGATE ST3000DM001	3	0.2672	1,175
4	HGST HDS5C3030ALA	3	0.0082	44,360

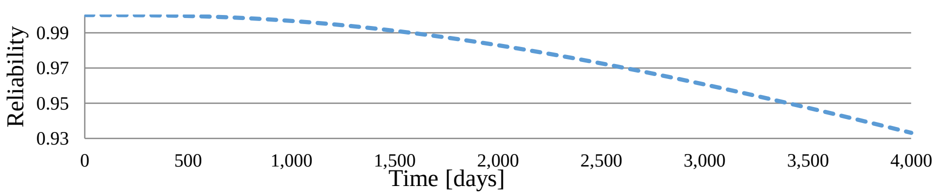


Fig. 3.3 Reliability function of the storage system

Reliability function $R(t)$ can also be used to find failure function $F(t)$ of the storage system as shown in (1.14). Time course of this function for the storage system is presented as the red solid line in Fig. 3.4. Based on its shape, we can find the same results as in the case of the reliability function, i.e., the unreliability of the storage system gradually increases with increasing time and, for example, after 1,572 days, it exceeds 0.01, what means that the system will become highly unreliable [35] according to the experts.

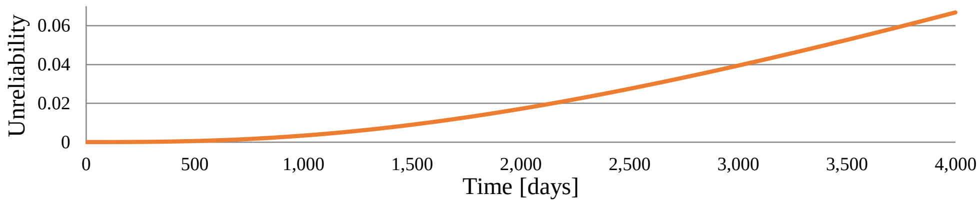


Fig. 3.4 Failure function of the storage system

By using functions $R(t)$ and $F(t)$, we can see how the system reliability and unreliability will change in time, but we are unable to conclude how the components are important for system operation. Therefore, we compute several importance measures for every component in the next step. All these computations can be done by using structure function (3.1) and logic differential calculus.

In the first step, we compute BI of each component. Because the system is coherent, we can compute BI using DPLDs and by their transformation into a probabilistic form according to (2.4). For this purpose, let us firstly compute DPLD for the first HDD of the storage system. This can be done by using formula (1.19) as:

$$\begin{aligned} \frac{\partial\phi(1\to 0)}{\partial x_1(1\to 0)} &= \overline{\big((0 \vee x_2) \vee x_3 \wedge x_4)\big)} \wedge \big((1 \vee x_2) \vee x_3 \wedge x_4\big) \\ &= \overline{(x_2 \vee x_3 \wedge x_4)} \wedge 1 = \bar{x}_2 \wedge (\bar{x}_3 \vee \bar{x}_4). \end{aligned} \tag{3.6}$$

This result implies that HDD 1 is critical for the system, i.e., its failure results in a failure of the system, if HDD 2 and at least one from HDDs 3 and 4 fails. Using the same procedure that was used to obtain the reliability from the structure function, we can transform Boolean formula (3.6) into probabilistic form, which agrees with the probability that HDD 1 is critical for the system, i.e., with the BI of HDD 1:

$$\mathrm{BI}_1 = 1 - p_2 - p_3 p_4 + p_2 p_3 p_4. \tag{3.7}$$

Using the same procedure as above, we can compute BI measures of the remaining components:

$$\begin{aligned} \mathrm{BI}_2 &= 1 - p_1 - p_3 p_4 + p_1 p_3 p_4, \\ \mathrm{BI}_3 &= p_4 - p_1 p_4 - p_2 p_4 + p_1 p_2 p_4, \\ \mathrm{BI}_4 &= p_3 - p_1 p_3 - p_2 p_3 + p_1 p_2 p_3. \end{aligned} \tag{3.8}$$

BI measures (3.7) and (3.8) do not depend on time, and they allow us to compute and compare importance of the components only for given values of the state probabilities of the components. However, the probabilities of individual states of the system components change as time flows, what implies that the importance of the components also changes over the time. If we want to investigate how these changes are, we have to compute time-dependent versions of BI measures by using (2.9). This can be done by using DPLD and transform it for the time dependent analysis. For example, application of this procedure results in the following formula for computation of time-dependent BI measure for HDD 1:

$$\mathrm{BI}_1(t) = 1 - P_2(t) - P_3(t)P_4(t) + P_2(t)P_3(t)P_4(t). \tag{3.9}$$

Since probability $P_i(t)$ is defined as a complement of lifetime distribution $Q_i(t)$ to value 1, for $i = 1{,}2{,}3{,}4$, we can write:

$$\begin{aligned} \mathrm{BI}_1(t) = Q_2(t) &- (1 - Q_3(t))(1 - Q_4(t)) \\ &+ (1 - Q_2(t))(1 - Q_3(t))(1 - Q_4(t)). \end{aligned} \tag{3.10}$$

In the similar way, we can obtain time-dependent versions of BI measures of the rest of the components of the storage system. Time courses of all these measures are depicted in Fig. 3.5. In the figure we can see that all the components of the system have similar importance at the beginning of the system operation but at time 500+ days, BI of HDD 1 is much greater than BI measures of the remaining HDDs, and its importance grows about four times faster than the importance of HDD 2. We can also conclude that the component with the least importance is HDD 4 and its BI does not change during the time. These results are quite reasonable because HDD 4 is connected in series with HDD 3, which means it influences system operation if HDD 3 is working. Since HDD 3 is very unreliable (its MTTF presented in Tab. 3.1 is very low), it is very likely this HDD is not functioning. Therefore, a failure of HDD 4 can result in system failure only with a small probability. Furthermore, since HDD 3 penalizes activity of HDD 4, which is the most reliable HDD according to MTTFs, importance of HDDs 1 and 2 grows. From these two HDDs, the most important is HDD 1 because if it fails, then it is very likely that the system fails. This results from the fact that a path composed of HDDs 3 and 4 (Fig. 3.2) is very unreliable since HDD 3 has little MTTF, and a path containing HDD 2 is less reliable than a path containing HDD 1 since MTTF of HDD 2 is about four times less than MTTF of HDD 1.

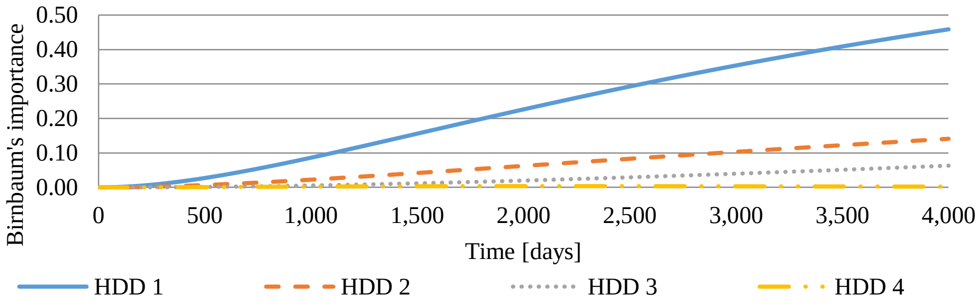


Fig. 3.5 Time-dependent BI measures for HDDs of the storage system

The BI measures presented in Fig. 3.5 show how the probabilities that a failure of individual components results in system failure changes over time. However, these measures do not consider reliability of components for which they are computed, i.e., BI of component i does not depend on lifetime distribution $Q_i(t)$. To avoid this problem, CI measures can be computed. As in the case of BI, static (without time) and dynamic (with time) version of CI can be computed. Time-dependent CI can be computed for HDD 1 based on (2.10) as:

$$\mathrm{CI}_1(t) = \mathrm{BI}_1(t)\frac{Q_1(t)}{F(t)} = \mathrm{BI}_1(t)\frac{Q_1(t)}{1 - R(t)}, \quad (3.11)$$

where $\mathrm{BI}_1(t)$ is time-dependent BI of HDD 1 computed in (3.9), $R(t)$ is reliability function (3.3) of the storage system, and $Q_1(t)$ is lifetime distribution of component i defining the probability that the component fails throughout interval $\langle 0, t \rangle$ given that it worked at time 0. After substituting all functions by their formula, we obtain the following result:

$$\mathrm{CI}_1(t) = 1, \quad (3.12)$$

which implies that CI of HDD 1 does not depend on time. This result agrees with our expectations because HDD 1 constitutes one path in reliability block diagram in Fig. 3.2 and, therefore, its repair surely results in system repair if we know that the system has failed. This corresponds to the meaning and usage of CI, because CI allows us to find components whose repair results in system repair with the greatest probability given that the system has failed.

The similar procedure can be used to find time-dependent CI measures for the rest of the components. Their time courses are shown in Fig. 3.6. From the graph, we can conclude that the most important components are HDDs 1 and 2, while a HDD with the least importance is HDD 4. This order of importance is similar to that obtained using the BI measures, but the values of CI measures are completely different from BI ones.

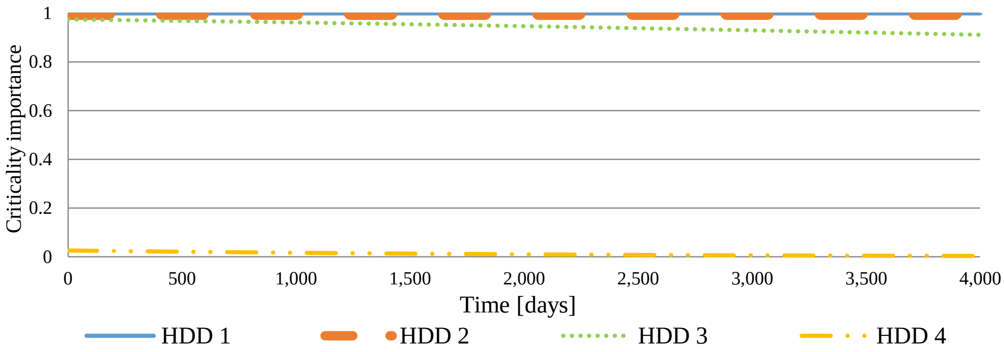


Fig. 3.6 Time-dependent CI measures for HDDs of the storage system

Until now the reliability analysis of the Storage system focuses on each component separately because it was necessary to compute how each component is important for the analysed system. Now we will also analyse how pair of HDDs can affect the system reliability. Using the computed structure and reliability functions, we can compute IMs for two HDDs to analyse how each HDD pair affects the storage system. For this purpose, we can use JRI and FI measures.

Firstly, we will compute JRI using reliability function. For example, by using (3.2) we can easily compute $\mathrm{JRI}_{1,2}$ for HDD 1 and HDD 2 as follows:

$$\mathrm{JRI}_{1,2} = \frac{\partial^2 R}{\partial p_1 \partial p_2} = -1 + p_3 p_4. \tag{3.13}$$

Since $p_3 p_4 \in \langle 0,1 \rangle$, the $\mathrm{JRI}_{1,2} \le 0$. Based on the meaning of JRI, this result implies that HDD 1 (HDD 2) is more important with respect to the system reliability when HDD 2 (HDD 1) is failed than when HDD 2 (HDD 1) is working. This is quite logical because HDDs 1 and 2 are arranged in parallel and, therefore, if one of them fails, then another one becomes more important. Using the same procedure, the JRI measures of the remaining pairs can be computed, i.e.:

$$\begin{aligned}
\mathrm{JRI}_{1,3} &= -p_4 + p_2 p_4, \\
\mathrm{JRI}_{1,4} &= -p_3 + p_2 p_3, \\
\mathrm{JRI}_{2,3} &= -p_4 + p_1 p_4, \\
\mathrm{JRI}_{2,4} &= -p_3 + p_1 p_3, \\
\mathrm{JRI}_{3,4} &= 1 - p_1 - p_2 + p_1 p_2.
\end{aligned} \tag{3.14}$$

Based on these results, it can be shown simply that all measures except $\mathrm{JRI}_{3,4}$ are nonpositive numbers. So the interaction between HDDs 1 and 3, 1 and 4, 2 and 3, and 2 and 4 can be interpreted same as in the case of interaction between HDDs 1 and 2 (formula (3.13)), i.e., if one of the HDDs in these pairs fails, then another HDD from the pair becomes more important. A special case is $\mathrm{JRI}_{3,4}$. Formula for computation of this measure can be rewritten as follows:

$$\mathrm{JRI}_{3,4} = 1 - p_1 - p_2 + p_1 p_2 = 1 - \Pr\{x_1 \vee x_2\}. \quad (3.15)$$

This implies that $\mathrm{JRI}_{3,4} \geq 0$. According to the meaning of JRI, this fact indicates that HDD 3 (HDD 4) is more important for system operation when HDD 4 (HDD 3) is functioning than when HDD 4 (HDD 3) fails. This result is also quite intuitive because HDDs 3 and 4 are arranged in series and, therefore, a failure of one of them causes that the second one cannot contribute to system reliability.

JRI measures (3.13) and (3.14) are computed regardless of time. If we want to investigate how interaction of two components evolves over time, we have to transform them into time-dependent versions. This can be done simply by replacing probabilities p_i with the time-dependent probability functions $P_i(t)$, for $i = 1,2,3,4$. Such a replacement causes that the JRI measures become time-dependent functions. For example, JRI (3.13), which measures interaction between HDDs 1 and 2 in the storage system, gets the next form:

$$\mathrm{JRI}_{1,2}(t) = -1 + P_3(t) P_4(t). \quad (3.16)$$

Using the same procedure, we can also obtain functions describing interactions between the remaining pairs of HDDs over time. These functions are depicted in Fig. 3.7.

In Fig. 3.7, we can firstly notice that only $\mathrm{JRI}_{3,4}$ is nonnegative during the whole time captured in the figure, while the other JRI measures are non-positive. This agrees with the results obtained in static analysis performed in (3.13) – (3.15). Secondly, we can see that the absolute values of all JRI measures except $\mathrm{JRI}_{1,4}$ increase during the whole time. This means that interaction between the components increases over time. The biggest increase is in absolute value of $\mathrm{JRI}_{1,2}$ and $\mathrm{JRI}_{1,3}$. In case of $\mathrm{JRI}_{1,2}$, this result can be explained as follows: HDDs 3 and 4 constitutes one path in the reliability block diagram depicted in Fig. 3.2. However, this path is high unreliable because HDD 3 is the least reliable HDD (its MTTF is much lower than MTTFs of the remaining HDDs). Therefore, there is a big chance this path fails as the first from the three paths depicted in Fig. 3.2. If this path fails, then the system can be functioning if and only if at least one of HDDs 1 and 2 is functioning. Therefore, a failure of one of these two HDDs causes that the second HDD becomes very important because it represents the last working path of the system. Similarly, the absolute value of $JRI_{1,3}$ increases very fast because HDD 2, which constitutes the middle path in the reliability block diagram depicted in Fig. 3.2, is quite unreliable and, therefore, there is a big chance that this path fails soon. However, the probability of this event is less than the probability that the path composed of HDDs 3 and 4 fails, therefore, the absolute value of this JRI increases slower over time than the absolute value of JRI of HDDs 1 and 2.

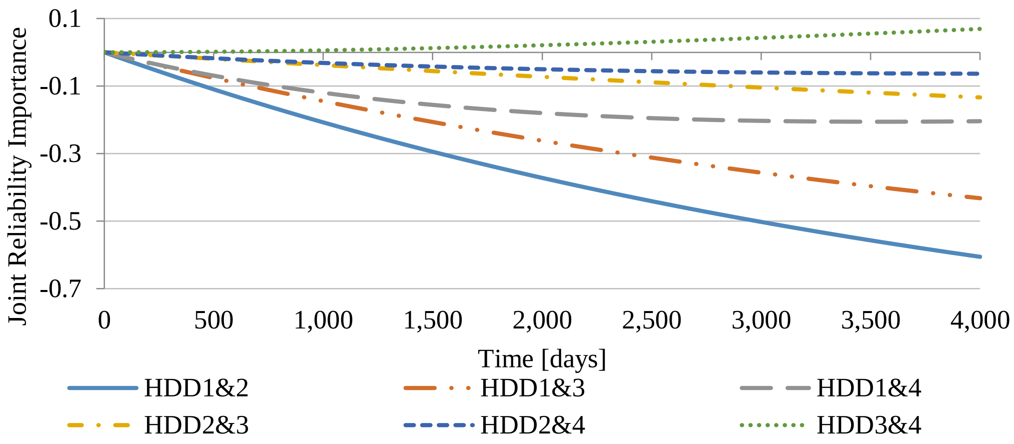


Fig. 3.7 Time-dependent JRI measures for pairs of HDDs

Next, the absolute values of $JRI_{2,3}$ and $JRI_{2,4}$ increase very slowly. This results from the fact that if one of the HDDs from the pairs of HDDs investigated by these measures fails, then the middle or the bottom path in Fig. 3.2 fails. However, these two paths are less reliable than the upper path containing HDD 1 (based on data presented in Tab. 3.1). This implies that a failure of one of these two paths causes that another path becomes more important, but its importance will be less than the importance of the upper path because the upper path has the biggest contribution to the system reliability.

A special case is $JRI_{1,4}$. The absolute value of this measure grows fast during the first 1,000 days, but then the speed of growth begins decreasing and after 3,000 days, it begins stagnating. This can be explained as follows. During the first 1,000 days, it is highly probable that all HDDs are functioning. This implies all three paths in the reliability block diagram in Fig. 3.2 are functioning and have a similar contribution to the system reliability. Therefore, a failure of one of HDDs 1 and 4 causes that one path fails. However, it is very probable that the remaining two paths will be functioning. Because of that, $\mathrm{JRI}_{1,4}$ has little values at the beginning of the time. Furthermore, since the probability of a failure of any HDD grows over time, the probability that the both remaining path will be functioning decreases and, therefore, $\mathrm{JRI}_{1,4}$ increases. However, after 3,000 days, the probability that the bottom path containing HDD 4 is functioning will be very low (it tends to 0) and, therefore, an effect of a failure of HDD 4 on importance of HDD 1 cannot grow after this time. The same is true for HDD 1.

Finally, the JRI of HDDs 3 and 4 grows very slowly over time. Based on the meaning of a nonnegative value of JRI, importance of one of these two HDDs for system operation increases over time when another HDD is functioning than when another HDD fails. This is quite logical because the probability that at least one of the upper and middle path in Fig. 3.2 is functioning decreases over time and, therefore, importance of HDD 3 (HDD 4) has to grow over time if HDD 4 (HDD 3) is functioning. On the other hand, the value of $\mathrm{JRI}_{3,4}$ grows very slowly because the probability that no path from the upper and middle is working is much less than the probability that the bottom path composed of HDDs 3 and 4 is working.

We would like to point out that changes in components reliability can lead only to change of the absolute value of JRI but not to the change of the JRI polarity. The JRI measures presented in Fig. 3.7 show how a failure of one HDD in a pair influences the importance of another HDD in the storage system. However, if we want to compute how simultaneous failure of both HDDs will affect the system, we have to use another measure. For this purpose, we introduced the FI [30]. This measure is defined based on a DPLD computed with respect to a vector of values. Now, we use it in importance analysis of the storage system.

Using DPLD (1.22), we can identify situations when a simultaneous failure of HDDs 1 and 2 results in a failure of the storage system, i.e.:

$$\frac{\partial\phi(1 \to 0)}{\partial(x_1, x_2)((1,1) \to (0,0))} = \overline{x_3} \vee \overline{x_4}. \tag{3.17}$$

This DPLD implies that a simultaneous failure of HDDs 1 and 2 results in system failure if at least one of HDDs 3 and 4 is failed. If we compute the probability that such situations occur, then we obtain the value of the FI, i.e.:

$$\mathrm{FI}_{1,2} = \Pr\left\{\frac{\partial\phi(1 \to 0)}{\partial(x_1, x_2)((1,1) \to (0,0))} = 1\right\} = 1 - p_3 p_4. \tag{3.18}$$

IM (3.18) allows us to compute the probability that a simultaneous failure of HDDs 1 and 2 results in a failure of the system. If we replace probabilities p_3 and p_4 by time-dependent functions $P_3(t)$ and $P_4(t)$ respectively, we can obtain the time-dependent version of this measure:

$$\mathrm{FI}_{1,2}(t) = 1 - P_3(t)P_4(t). \tag{3.19}$$

Based on this formula, we can quantify consequences of a simultaneous failure of HDDs 1 and 2 on the operation of the storage system over time. This function is depicted in Fig. 3.8 as the blue dotted line. As we can see, this measure grows over time. This result can be explained as follows: HDDs 1 and 2 are arranged in parallel, and they correspond to two paths in the reliability block diagram in Fig. 3.2. Simultaneously, HDDs 3 and 4 constitutes another path. Since the reliabilities of HDDs decrease as time flows, the probability that the path composed of HDDs 3 and 4 is functioning has to decrease over time. This implies there is a little probability that the path composed of HDDs 3 and 4 will be working at a late stage of system operation. Therefore, a simultaneous failure of HDDs 1 and 2 results in system failure at a late stage with a greater probability than in the early phase. Using the same procedure as above, we can compute the FI for the other pairs of HDDs:

$$\begin{aligned}
\mathrm{FI}_{1,3}(t) &= \mathrm{FI}_{1,4}(t) = 1 - P_2(t), \\
\mathrm{FI}_{2,3}(t) &= \mathrm{FI}_{2,4}(t) = 1 - P_1(t), \\
\mathrm{FI}_{3,4}(t) &= 1 - P_1(t) - P_2(t) + P_1(t)P_2(t).
\end{aligned} \tag{3.20}$$

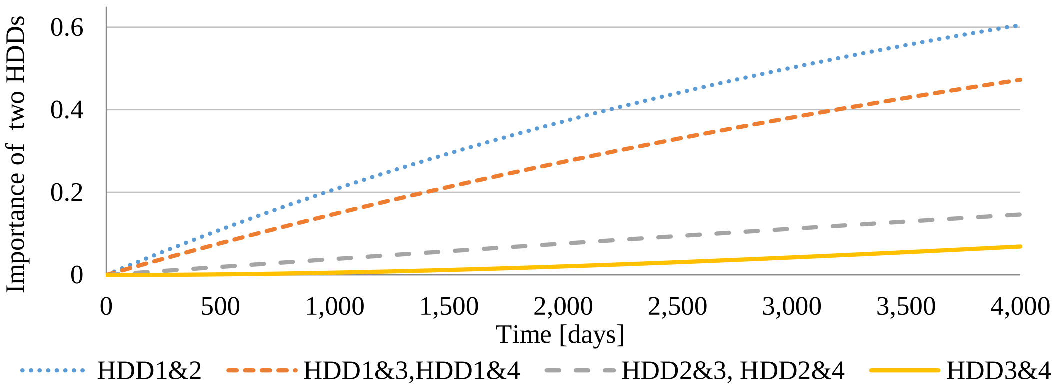


Fig. 3.8 Time-dependent FI measures for pairs of HDDs

Time courses of these measures are shown in Fig. 3.8. As we can see, the relative importance of a simultaneous failure of the pairs of HDDs does not change over time. Therefore, we can conclude that the pairs of components, whose simultaneous failure has the greatest influence on system activity, are pairs containing HDD 1. This result agrees with the fact that a simultaneous failure of HDD 1 and another HDD results in a failure of the most reliable path in Fig. 3.2 (the path containing HDD 1) and some other path. Similarly, if HDD 1 does not fail, then the most reliable path will be working. Because of that, a simultaneous failure of two HDDs different from HDD 1 has less impact on system operation.

3.1.2 Case Study for Drone Fleet

We will continue with time dependent reliability analysis based on the structure function and time-dependent BI and CI measures with the use of DPLDs on the drone fleet that is depicted using RBD in Fig. 3.9.

Analysed drone fleet is a part of the complex surveillance monitoring system. There are three types of components in this fleet. First type is control unit (CU) that control and manage n_d drones in fleet. CU is the most crucial part of the fleet, because if this component fails, then all drones cannot perform any scheduled tasks. Second component type is main drone (MD). This type represents all k_d main drones in the fleet that are used to perform scheduled tasks, which are set by CU. Last component type represents $n_d - k_d$ redundant drones (RD) in the fleet. Those drones are back up for the main drones. This means that if some main drone fails, available redundant drone will continue to perform its duties.

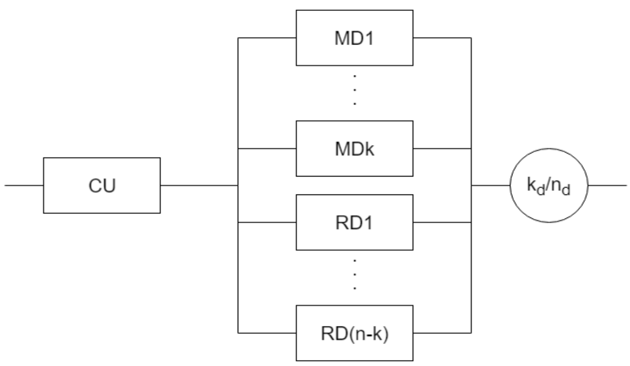


Fig. 3.9 RBD for the drone fleet

The main fleet as an analysed system can be in two states. If CU is functioning and at least k_d drones are functioning, then the drone fleet can perform all scheduled tasks and, therefore, the system is functioning. If those conditions are not met, then the system is failed. It is needed to point out that in reliability analysis we will presume that CU and all drones are irreparable and at start all components are in working states. We will not take the recharge time for each drone into account because it is irrelevant for our analysis.

We will consider the following settings for drone fleet. CU can control up to 5 drones at a time, which means that $n_d = 5$. Because we want to have redundant drones in the fleet and we need at least two drones to be working, then the k_d can have three values that are 2, 3, and 4. There are two possible configurations for drone fleet. Either all drones are the same type or the main drones have same type and the redundant drones have same type.

According to the RBD and system description, we can conclude that this system can be seen as serial system with two modules, where first module is CU and the second module is k_d-out-of-5 system representing all drones in fleet for $k_d \in \{2,3,4\}$.Therefore, the structure function for this system has the following form:

$$\phi(x_1, x_2, x_3, x_4, x_5, x_6) = x_1 \wedge \bigvee_{\mathrm{S}_i \in \mathrm{S}_{\frac{k_d}{5}}} \bigwedge_{j \in \mathrm{S}_i} x_j. \tag{3.21}$$

This structure function is defined for all $k_d \in \{2,3,4\}$. For example, if $k_d = 3$, then the structure function (3.21) has the following form:

$$\begin{aligned}\phi(x_1, x_2, x_3, x_4, x_5, x_6) \\ = x_1 \wedge (\ x_2 \wedge x_3 \wedge x_4 \ \vee\ x_2 \wedge x_3 \wedge x_5 \ \vee\ x_2 \wedge x_3 \wedge x_6 \ \vee\ x_2 \\ \wedge x_4 \wedge x_5 \ \vee\ x_2 \wedge x_4 \wedge x_6 \ \vee\ x_2 \wedge x_5 \wedge x_6 \ \vee\ x_3 \wedge x_4 \wedge x_5 \ \vee\ x_3 \\ \wedge x_4 \wedge x_6 \ \vee\ x_3 \wedge x_5 \wedge x_6 \ \vee\ x_4 \wedge x_5 \wedge x_6).\end{aligned} \tag{3.22}$$

As next step in our analysis, we will compute time-dependent reliability measures using obtained structure function. In our analysis, we will assume that all system components are unrepairable and stochastically independent, and we will use the reliability measure Mean Time to Failure (MTTF) that represents average time, in which the system fails. All MTTFs needed for reliability analysis are obtained from [36] and their values are shown in Tab. 3.2. Those values will be used to compute parameters for lifetime distributions of system components. For them, we will use exponential and Weibull distribution [36]–[38].

Tab. 3.2 MTTF for each component of the drone fleet

Type of component	**MTTF [hours]**
Control unit	500,000.00
Basic Drone	26,809.65
Advanced Drone	35,423.31

Firstly, we will analyse system settings in which all drones are same, therefore, they have the same lifetime distribution. In this case, we will focus on two major decisions. First decision is to choose the k_d. From definition of k-out-of-n system configuration we can conclude, that this drone configuration will be most reliable for the $k_d = 2$, but having only two drones in main fleet will result in limitation of the concurrent tasks performance. Ideal k_d for concurrent tasks performace is 4, but the fleet reliability will be considerably decreased in this case. Therefore, we choose to compute time-dependent reliability measure for each k_d according to the following formula obtained from structure function:

$$P_{CU}(t) * \sum_{\mathrm{S}_i \in \mathrm{S}_{k_d/n}} \prod_{j \in \mathrm{S}_i} P_i(t) \prod_{j \in \mathrm{N}-\mathrm{S}_i} (1 - P_i(t)), \tag{3.23}$$

where $P_{CU}(t)$ represents probability function of CU functioning in time t and $P_i(t)$ represents probability function of each drone to be in functioning state in time t. We will use exponential distribution to characterize CU and Drones lifetime distribution with $\lambda = 1/\mathrm{MTTF}$ for CU and basic drone. The result can be seen in graph on Fig. 3.10, in which the axis X represents the time in hours and the axis Y represents the values of the reliability function. From this graph we can conclude that if top priority for drone fleet is reliability, then the $k_d = 2$. If the top priority is number of concurrent tasks, then the $k_d = 4$. As for $k_d = 3$, this configuration represents appropriate compromise between reliability and the number of the concurrent tasks. Therefore, we choose $k_d = 3$ for our next analysis.

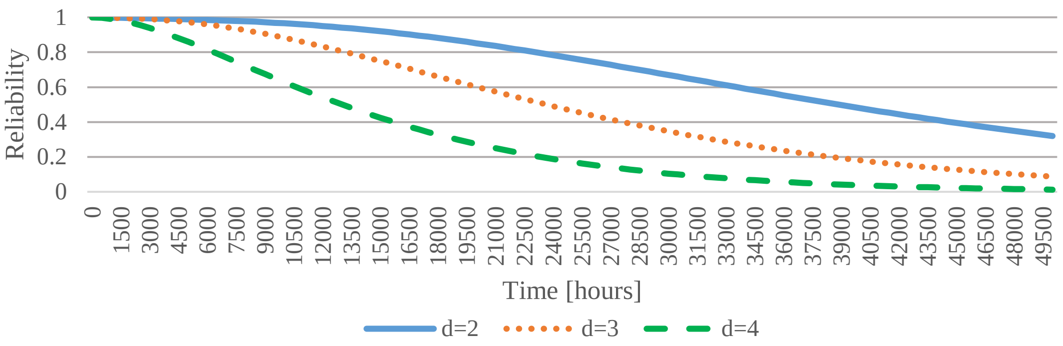


Fig. 3.10 Reliability function of the drone fleet for all values of k_d

In the second step of the homogenous analysis, we will analyse how the change in MTTF for drones will affect the system reliability. In Tab. 3.2 we have two values of MTTF for drones. The first one is for basic drones and the second one represents more advanced and pricey drone. Therefore we choose to analyse, how the change in the MTTF affects the system reliability. This will be helpful for buying drones, that are not that pricey, but are reliable enough for our needs. There were 10 values of MTTF, that we consider for this analysis, where the first one was MTTF for basic drone, the last one was for the more reliable drone and all MTTFs between those two values are evenly distributed values between basic and more reliable drone. Result of our analysis can be seen on a 3D graph shown in Fig. 3.11, in which time in hours is on the X axis, values of the reliability function are on the Y axis, and different MTTFs are on the Z axis. It is possible to see that in this graph the gradual increase in reliability is starting to be more visible in 4,000 hours after the system start point and in 50,000 hours the difference between reliability of basic and more reliable drone is around 6%.

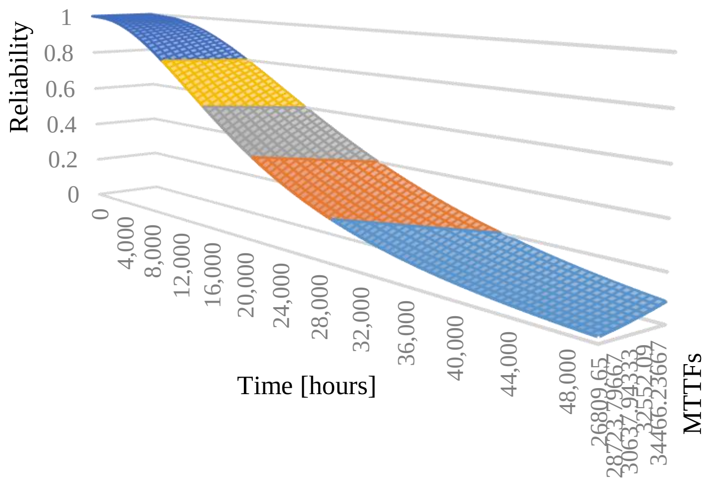


Fig. 3.11 Reliability function of the system for different values of MTTF

In the second part of the reliability analysis we will be focusing on the heterogeneous types for drones in drone fleet. We will assume that three drones will be more reliable drones with exponential lifetime distribution and remaining two drones will be basic drones with Weibull lifetime distribution with shape parameter 0.8. Using different types of drones in fleet is not a problem for reliability analysis using structure function, because we can easily use (3.23), where we just use correct $P_i(t)$ for each drone. The resulting reliability function is shown on graph in Fig. 3.12, where x-axis represents time in hours and y-axis represents the value of the reliability function. When we compare this result with the homogenous setting, we can easily see that in this case the value of the reliability function is decreasing slightly faster than in the homogenous settings but in more than 10,500 hours the decreasing starts vanishing. This is caused by Weibull shape parameter because its value is below 1 and that means that the fault rate will be decreasing over longer time period.

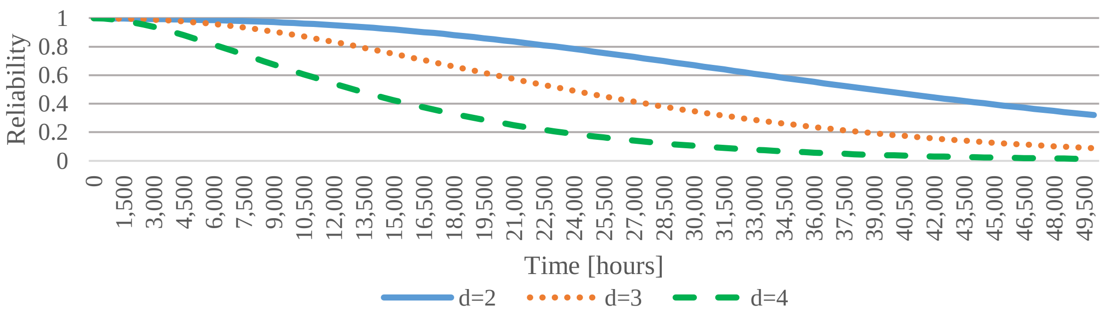


Fig. 3.12 Reliability function of the heterogeneous drone fleet for each k_d

As a next step in heterogeneous reliability analysis we will compute Birnbaum's IM and Criticality IM for each drone (numbered from 1 to 5) by using structure function and

DPLDs as shown in (2.9), and therefore we will understand how each drone is important for system reliability.

The Birnbaum's IM time course for each drone can be seen in Fig. 3.13. From them we can see, at the beginning the most crucial components for the drone fleet are the basic type drones and in around 22,000 hours are all components equally important for the system reliability. From this point on, the most important components are advanced drones. This is mostly caused by their MTTFs.

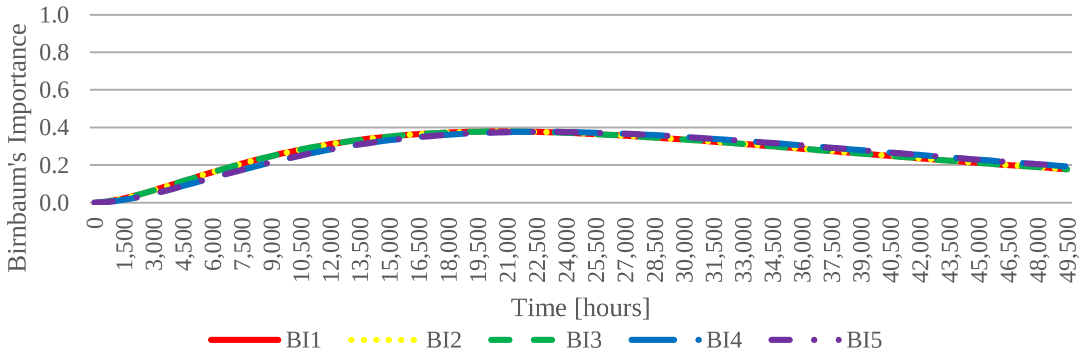


Fig. 3.13 Time course of BIs for the heterogeneous drone fleet

After that we compute the Criticality IM for each drone by using (2.10) and its time course can be seen in Fig. 3.14. Here we can see, that failure of the advanced drones has the highest influence on the system failure on the beginning and then it rapidly decreases for 500 hours and after this point their influence slowly decreased. On the other hand, basic drones have less influence at the beginning, but then their influence start to rise rapidly for 500 hours. Then their influence gradually decreases.

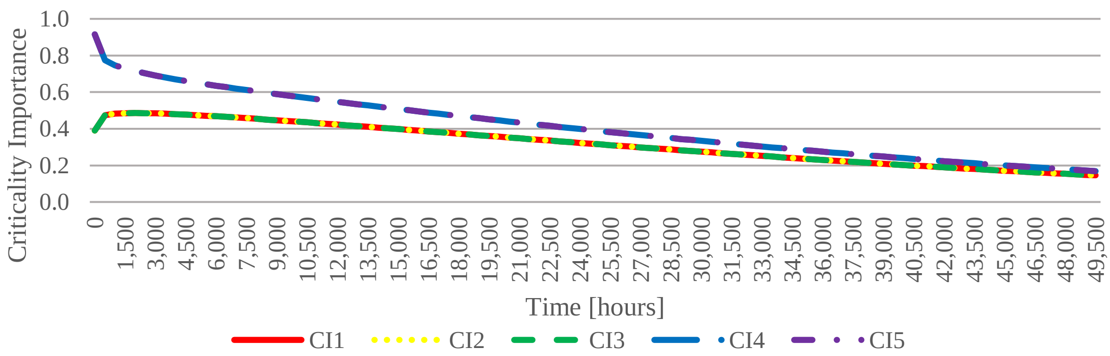


Fig. 3.14 Time course of CIs for the heterogeneous drone fleet

3.1.3 Case Study for Surveillance System

Lastly, we will show how the time-dependent reliability analysis based on the structure function can be performed and time-dependent BI and CI measures can be computed with the use of DPLDs on the surveillance system. In this section, we will show how reliability analysis using structure function and DPLD can be performed using structure function of surveillance system that is shown in Fig. 3.15. This system is composed of four cameras that are connected to the network with one network switch and network-attached storage (NAS) with two hard disks. All four cameras are used for monitoring the same square shaped room. Records from all four cameras are send through the network switch to the NAS, where are those records duplicated and stored on two different hard drives. In order to provide the successful room monitoring, we need to have at least one camera in working state, at least one hard drive in NAS must be in working state and network switch must be in working state. In our reliability analysis, we will assume that the network cables are perfectly reliable and we will not take their reliability into account in our reliability analysis.

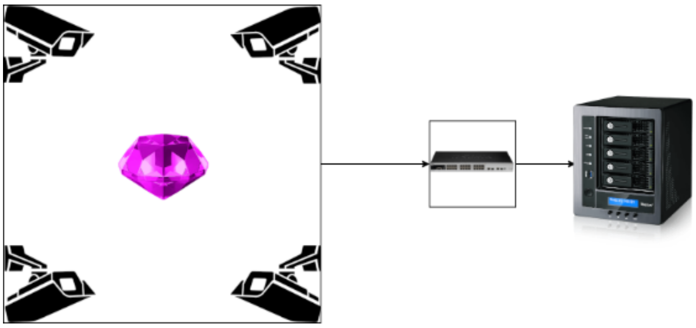


Fig. 3.15 Illustration of the surveillance system

According to the description of the surveillance system, we will define this system as a binary state system. The working state of the analysed system will correspond with the situation, in which the room is successfully monitored and if the room cannot be successfully monitored, then the system is failed. As for the system components, they are in the working state if they can perform their tasks for room monitoring and are in failed state otherwise. According to this system description, we can make the RBD for this system and it can be seen in Fig. 3.16.

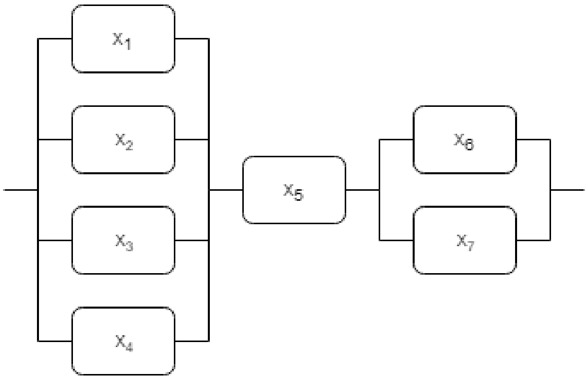


Fig. 3.16 RBD of the surveillance system

Taking the description of the surveillance system and its RBD into account, the structure function for the analysed system is as follows:

$$\phi(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (x_1 \vee x_2 \vee x_3 \vee x_4) \wedge x_5 \wedge (x_6 \vee x_7), \tag{3.24}$$

where x_1, x_2, x_3, x_4 are Boolean variables representing the state of all cameras in the room, Boolean variable x_5 represents the state of the network switch and Boolean variables x_6 and x_7 represents the state of the hard disk drives in the NAS.

As a next step, we obtain reliability from (3.24) which has the following form:

$$R = p_5 - q_1 * q_2 * q_3 * q_4 * p_5 - p_5 * q_6 * q_7 + q_1 * q_2 * q_3 * q_4 * p_5 * q_6 * q_7, \tag{3.25}$$

where p_i or q_i for $i = 1,2, \dots ,7$ represents the probability, that the i-th component is working or failed. If we want to take time into account for (3.25), we can simply replace the probabilities with lifetime distributions. Lifetime distribution for each component in this system agrees with the exponential distribution, where $\lambda_i = 1/\mathrm{MTTF}_i$ for $i = 1,2, \dots ,7$. MTTF for each component can be seen in Tab. 3.3.

Tab. 3.3 MTTF for each component of the surveillance system

Component	**Component data**	
	Component name	***MTTF [years]***
1-4	TK-C9200E(EX)	10.61
5	RUGGEDCOM M969	97.1
6	ST8000NM0055	92.206
7	HUH728080ALE600	117.961

In order to see how each component is important for system reliability, we will compute Birnbaum's IM and Criticality IM by using structure function and DPLDs according to (2.9).

Firstly, we compute the Birnbaum's IM. Its time course for each component can be seen in Fig. 3.17. As we can see, at the beginning, the most crucial component for the system is component 5. This can be explained by its placement in the system because its failure will surely result in the system failure. However, as we can see from Fig. 3.17, its importance decreases in around 4 years to 50 years quite rapidly. This is due to the small MTBF of the components 1, 2, 3, and 4. From start, their importance raises rapidly until the time reaches around 32 years. After this point, their importance slowly decreases. As for components 6 and 7, their Birnbaum's IM is small. This is caused by their high MTTFs and their parallel placement in the system.

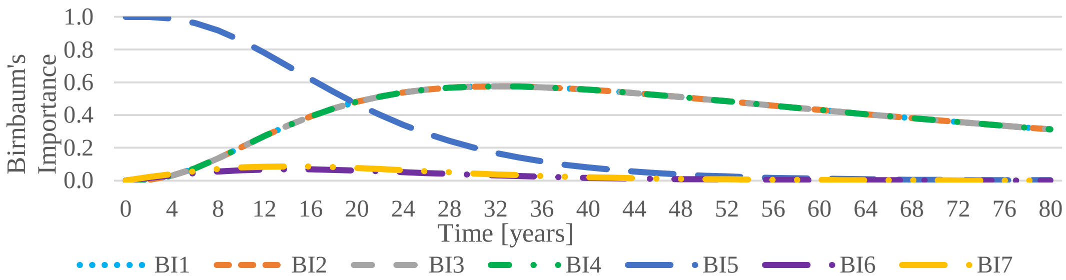


Fig. 3.17 Time course of BIs for the surveillance system

After that we compute the Criticality IM and its time course can be seen in Fig. 3.18. Here we can see, that failure of the component 5 has the highest influence on the system failure at the beginning and then its influence starts to decrease. On the other hand, components 1, 2, 3, and 4 have almost none influence at the beginning, but then their influence start to rise rapidly until 19 years. Then their influence gradually decreases. As for the components 6 and 7, their influence is really small thanks to their great durability and their placement in the system.

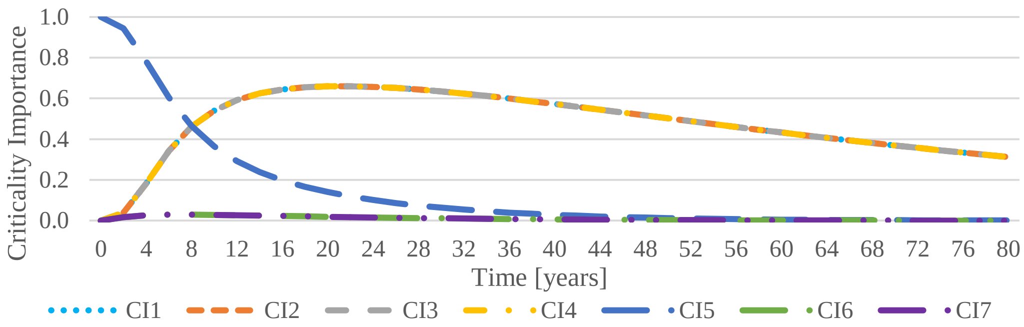


Fig. 3.18 Time course of CIs for the surveillance system

3.2 Case Studies Based on New DPLDs for Survival Signature

In this section, we will demonstrate the usage of logic differential calculus for survival signature on four selected systems. Those systems are series-parallel system, where we will depict the computation and usage of the proposed DPLDs, data storage system used in 3.1.1, series system with bridge topology and the inside mechanism of the hydro power plant.

3.2.1 Case Study for Series-Parallel System

First system that is depicted by Reliability Block Diagram in Fig. 3.19 is series-parallel system that is composed of 5 components, whose working states are represented by Boolean variables x_1, x_2, x_3, x_4, x_5 and components represented by variables x_1, x_2 have same type (type 1) and x_3, x_4, x_5 have same type (type 2). This is shown with label in top left corner and different colours in Reliability Block diagram in Fig. 3.19 (green – type 1 and blue – type 2). The structure function representing this system has following form:

$$\phi(x_1, x_2, x_3, x_4, x_5) = (x_1 \vee x_2 \wedge x_3) \wedge (x_4 \vee x_5). \qquad (3.26)$$

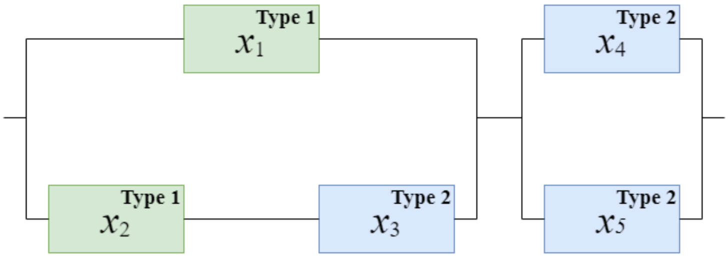


Fig. 3.19 Reliability block diagram of the series-parallel system

$$\Phi(l_1, l_2) = \left[\prod_{k=1}^{2} \binom{n_k}{l_k}^{-1}\right] * \sum_{x \in S_{l_1,l_2}} \phi(\boldsymbol{x}). \tag{3.27}$$

The flow-diagram for the calculation of system signature is shown in Fig. 3.20. Survival Signature for this system with the structure function (3.26) can be seen in Tab. 3.4.

Tab. 3.4 Survival signature for analysed system represented by (3.26)

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1, l_2)$
0	0	0
0	1	0
0	2	0
0	3	0
1	0	0
1	1	0.333333
1	2	0.833333
1	3	1
2	0	0
2	1	0.666667
2	2	1
2	3	1

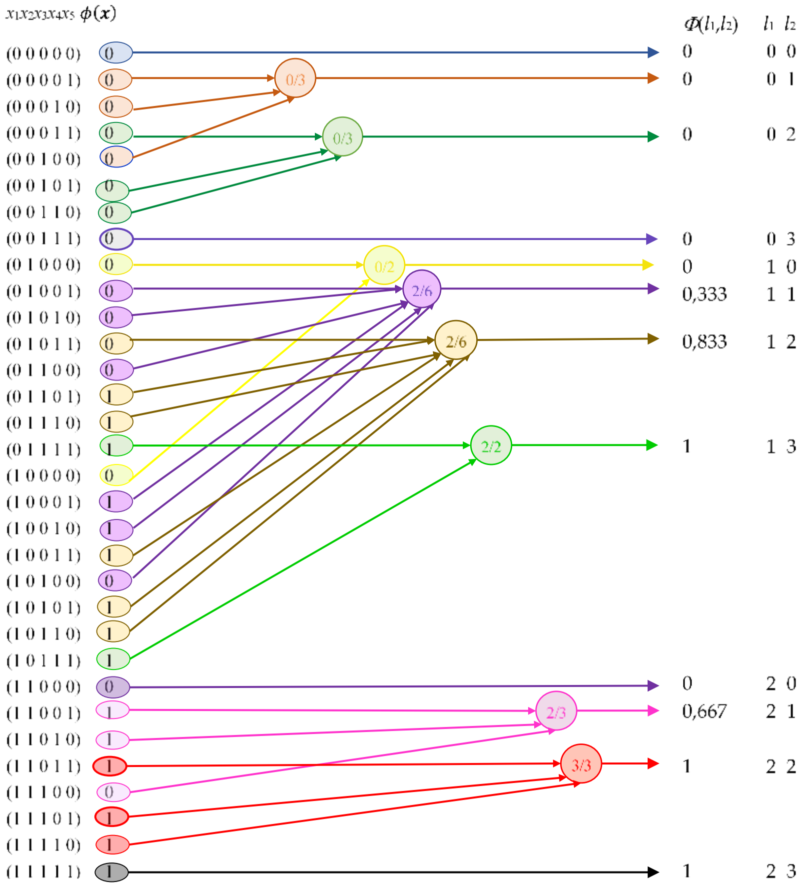


Fig. 3.20 Flow-diagram for survival signature of structure function (3.26)

The derivatives (2.11) allow us to indicate system states for which the breakdown of one of components of fixed type causes the system failure for indicated numbers of working component of every types. The considered system has components of two types. The set of components of the first type includes two components. The set of the second types consists of three components. We can consider two possibilities for the system failure depending on the breakdown of component of the first type that are indicated by derivatives $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(2\to1)}$ and $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(1\to0)}$. The derivative $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(1\to0)}$ allows us to indicate system failure if one working component of the first type will breakdown when other was faulted. The possibility of the system failure depending of the breakdown of one of two working components of this type is investigated by the derivative $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(2\to1)}$. From values of the $\mathrm{SI}^{\downarrow}_{1,2}$ and $\mathrm{SI}^{\downarrow}_{1,1}$ we can conclude, that the failure of the component of type 1 influence the system functionality more when there is only one working component of type 1. We have to consider three derivatives for the second type of components because the system failure can be resulted by the breakdown of one component if: only one component is working ($\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(1\to0)}$), two components of three are working ($\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(2\to1)}$), all components are working ($\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(3\to2)}$). From values of the $\mathrm{SI}_{2,3}^{\downarrow}$, $\mathrm{SI}_{2,2}^{\downarrow}$ and $\mathrm{SI}_{2,1}^{\downarrow}$ we can conclude, that the failure of the component of type 2 influence the system functionality more when there is two and one working component of type 2. The first DPLDs and $\mathrm{SI}_{k,a}^{\downarrow}$ for the first and second type of component are shown in Tab. 3.5. The flow-diagram in Fig. 3.21 illustrates the calculation of these derivatives.

Tab. 3.5 The first DPLDs and SI for survival signature of structure function (3.26) for components of the first and second types

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1,l_2)$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(2\to1)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(1\to0)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(3\to2)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(2\to1)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(1\to0)}$
0	0	0	-	-	-	-	-
0	1	0	-	-	-	-	0
0	2	0	-	-	-	0	-
0	3	0	-	-	0	-	-
1	0	0	-	0	-	-	-
1	1	0.333333	-	1	-	-	1
1	2	0.833333	-	1	-	1	-
1	3	1	-	1	1	-	-
2	0	0	0	-	-	-	-
2	1	0.666667	1	-	-	-	1
2	2	1	1	-	-	1	-
2	3	1	0	-	0	-	-
$\mathrm{SI}_{k,a}^{\downarrow}$			0.5	0.75	0.333333	0.666667	0.666667

For example, the derivative $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(1\to0)}$ for $l_1 = 1$ means the system failures if one of two working components of the second type breakdowns and one component of the first type is working. For the $l_1 = 0$ this derivative has value 0 that means absent of any changes in the system states if the one of two working components of the second type breakdowns and all components of the first type are fault ($l_1 = 0$). But need to take into account that the any changes does not mean the system functioning, because the system can be failure before this component breakdown. The value 1 for the derivatives of this types means the system failure is possible.

The derivatives (2.13) are generalisation of the derivatives (2.11) and allow us to define system states for which the breakdown of one of components of fixed type causes the system failure for fixed number of working components of other types. It is possible to compute two derivatives for this system $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}$ and $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2\downarrow}$ and their values and $\mathrm{SI}_k^{\downarrow}$ are shown in Tab. 3.6. From $\mathrm{SI}_1^{\downarrow}$ and $\mathrm{SI}_2^{\downarrow}$ is possible to see that the failure of the component of type 1 influence the system functionality more that type 2. The flow-diagram in Fig. 3.22

illustrate these derivatives calculation according to (2.13). Note that the derivatives in Tab. 3.6 can be calculated as the merge of derivatives in Tab. 3.5 for every variable according to (2.14).

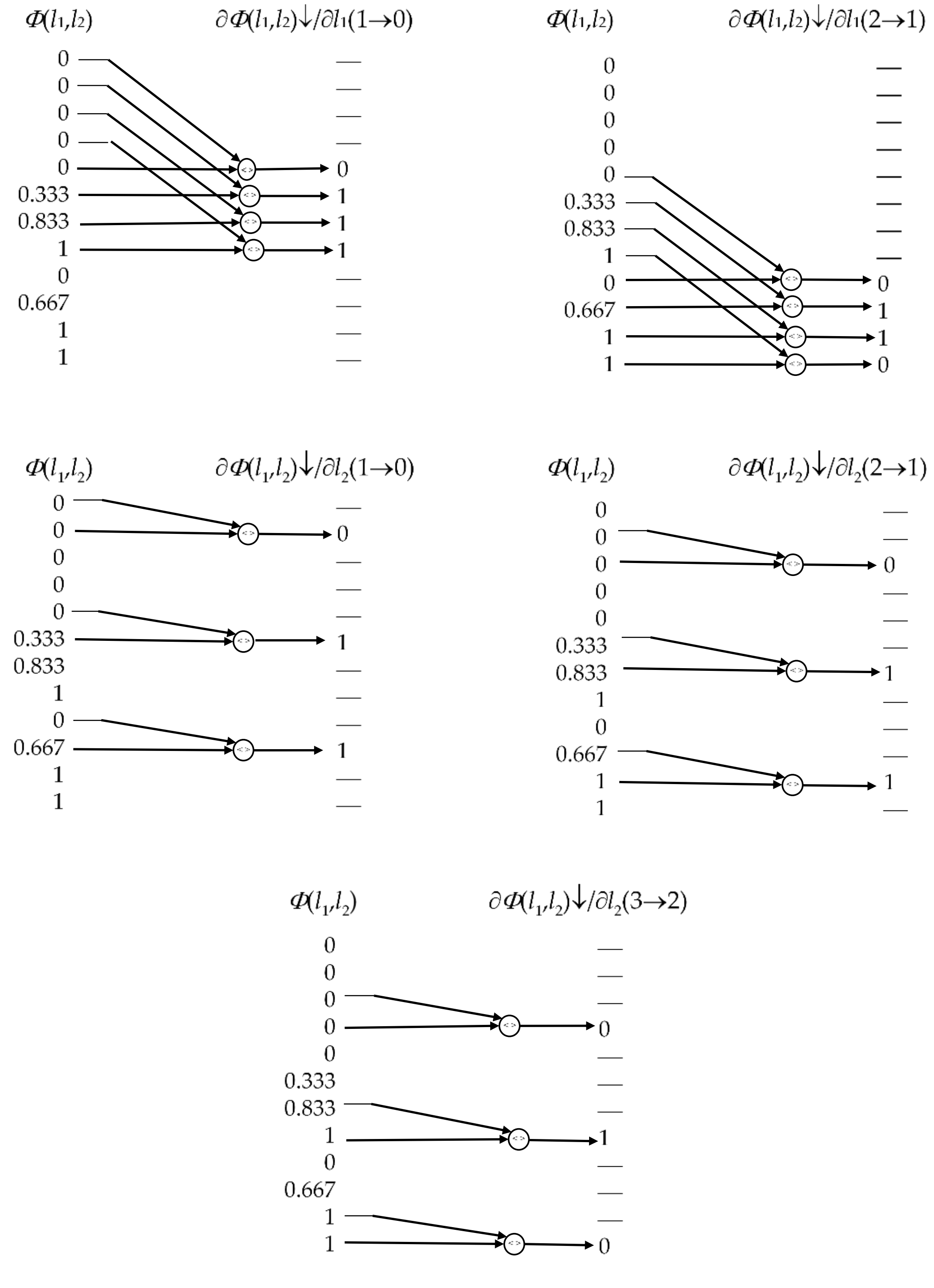


Fig. 3.21 Flow-diagram for the first DPLDs for survival signature of structure function (3.26)

Tab. 3.6 The second DPLDs and SI for survival signature of structure function (3.26) for components of the first and second types

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1, l_2)$	$\frac{\partial\Phi(l_1, l_2)\downarrow}{\partial l_1\downarrow}$	$\frac{\partial\Phi(l_1, l_2)\downarrow}{\partial l_2\downarrow}$
0	0	0	-	-
0	1	0	-	0
0	2	0	-	0
0	3	0	-	0
1	0	0	0	-
1	1	0.333333	1	1
1	2	0.833333	1	1
1	3	1	1	1
2	0	0	0	-
2	1	0.666667	1	1
2	2	1	1	1
2	3	1	0	0
$SI_k^{\downarrow}$			0.625	0.555556

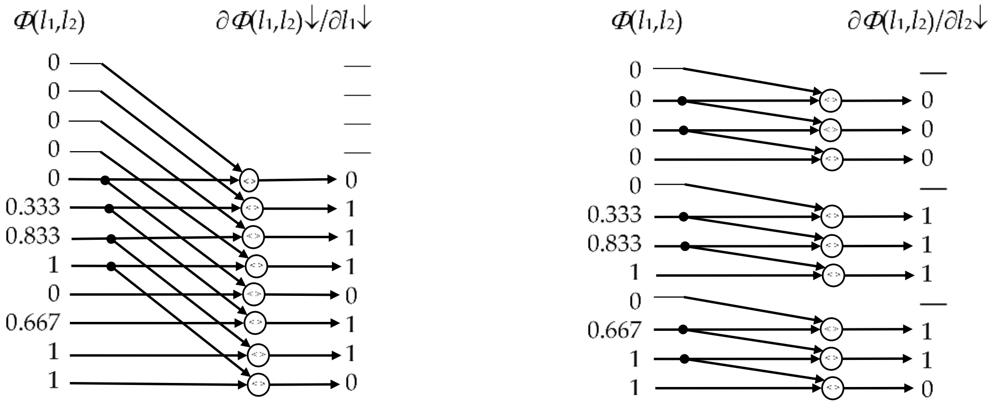


Fig. 3.22 Flow-diagram for the second DPLDs for survival signature of structure function (3.26)

As an example, according to the non-zero values of the derivative $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}$ can be consider next scenarios of the system failure:

- The breakdown of a single working component of two components of the second type can cause the system failure if one, two or three components of the second type are working;
- The breakdown of one component out of two working components of the first type can cause the system failure if one or two components of the second type are working.

The third DPLD for survival signature of structure function allows us to measure the system failure depending of the breakdown of one of components of the fixed type (Tab.

3.7). The non-zero values of these derivatives indicate the failure of the system depending on the component fault and shown the probability of this failure (Fig. 3.23).

Tab. 3.7 The third DPLDs and SI for survival signature of structure function (3.26) for components of the first and second types

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1,l_2)$	$\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_1\downarrow}$	$\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_2\downarrow}$
0	0	0	-	-
0	1	0	-	0
0	2	0	-	0
0	3	0	-	0
1	0	0	0	-
1	1	0.333333	0.333333	0.333333
1	2	0.833333	0.833333	0.5
1	3	1	1	0.166667
2	0	0	0	-
2	1	0.666667	0.333333	0.666667
2	2	1	0.166667	0.333333
2	3	1	0	0
$SI_k^{\Downarrow}$			0.333333	0.222222

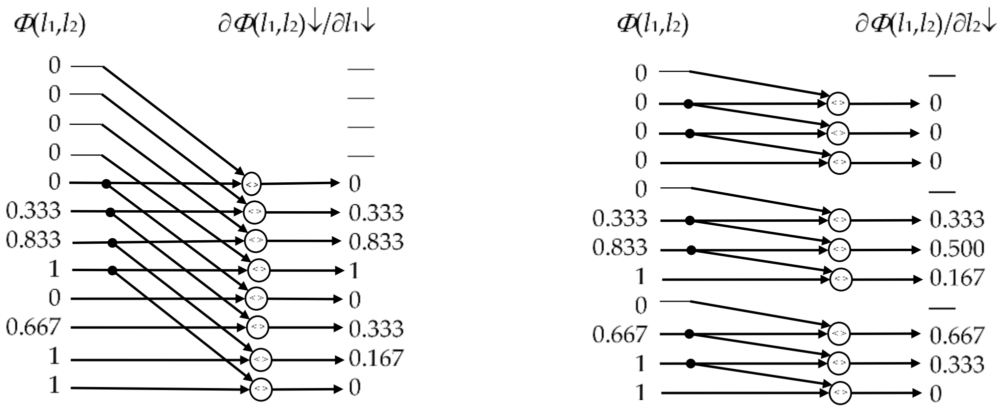


Fig. 3.23 Flow-diagram for the third DPLDs for survival signature of structure function (3.26)

For example we will consider the non-zero values of the $\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_1\downarrow}$. This derivative has 5 non-zero elements corresponding to $\Phi(1,1) = 0.333$, $\Phi(1,2) = 0.833$, $\Phi(1,3) = 1$, $\Phi(2,1) = 0.333$ and $\Phi(2,2) = 0.167$. These values have been computed as the difference of the survival signature of structure function value according to (2.16). But in the same time these derivatives can be calculated according to (2.17) based on the values of the DPLD of the structure function (3.26) (Fig. 3.24). This calculation shows that the values of the derivatives (2.17) depend on the number of the system states for which the breakdown of the components of fixed type cause the system failure according to values of $\frac{\partial\phi(1\to 0)}{\partial x_1(1\to 0)}$ and $\frac{\partial\phi(1\to 0)}{\partial x_2(1\to 0)}$. We consider the derivatives with respect to variables x_1 and x_2 because these variables interpret the first and second components states and these components of the first type.

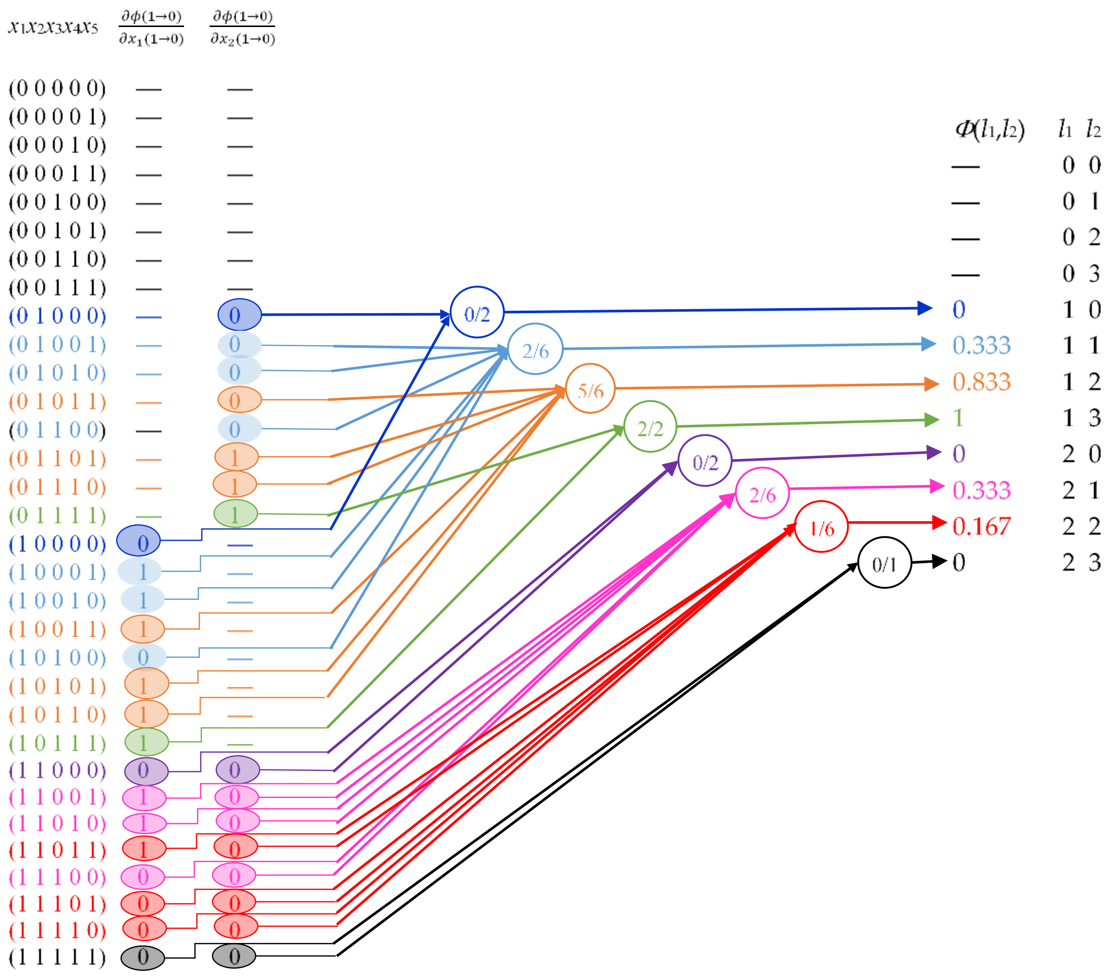


Fig. 3.24 Flow-diagram for the third DPLDs for survival signature of structure function (3.26)

For example, consider the calculation of the value of the third DPLD $\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_1\downarrow}$ for $l_1 = 1$ and $l_2 = 0$ that has value 0. In this type of DPLD the system states with one functioning component of the first type should be considered (01000 and 10000). The DPLDs $\frac{\partial\phi(1\to 0)}{\partial x_1(1\to 0)}$ and $\frac{\partial\phi(1\to 0)}{\partial x_2(1\to 0)}$ have values 0 for this state (Fig. 3.24). The zero values of these derivatives mean that the system does not fail depending on the breakdown of the first or second components. But it is needed to point out that it does not mean that the system functioning: the system can be no-functioning before the component state change (as is in this case).

Consider next value of the $\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_1\downarrow}$ for $l_1 = 1$ and $l_2 = 1$. This value is calculated based on 3 values of DPLD $\frac{\partial\phi(1\to0)}{\partial x_1(1\to0)}$ and 3 values of DPLD $\frac{\partial\phi(1\to0)}{\partial x_2(1\to0)}$. Thanks to the fact that only two are non-zero, it is possible to see that there are only two system states among all possible 6 states of the one functioning component of the first type and one functioning component of the second type for which the breakdown of component of the first type results the system failure. This proportion for next value ($l_1 = 2$ and $l_2 = 1$) is 5 out of 6. From $\mathrm{SI}_1^{\Downarrow}$ and $\mathrm{SI}_2^{\Downarrow}$ that are shown in Tab. 3.7 is possible to see that the failure of the component of type 1 influence the system functionality more that type 2.

Therefore, we can propose the interpretation of the third DPLD for survival signature of structure function:

- This derivative shows proportion of the system states among of all possible states of fixed numbers of functioning components of every type for which the breakdown of one specified component cause the system failure.
- The probability of the system failure for indicated system state by numbers of functioning components of every type if one specified component breakdowns.

3.2.2 Case Study for Storage System

Next system, that will be analysed by proposed derivations and survival signature is the storage system (Fig. 3.1) described in 3.1.1. This specific system is interesting thanks to the fact, that each component (HDD) has different type. Therefore, the meaning of the Boolean variables x_1, x_2, x_3, x_4 whose represent the state of the system components will change to the number of working components (either 1 or 0) of type $k \in \{1,2,3,4\}$ and the value of the survival signature will be either 1 or 0 depending on the state of the system for given number of working components of specific type. This means that the survival signature for this system is identical with structure function, and this can be seen in left part of Tab. 3.8. As for the proposed DPLDs, they all have the same value for each type and this value corresponds with the DPLD (1.19) as can be seen in right part of Tab. 3.8. Same can be said about SI. This can be explained by the fact that in this system each component has distinct type and the system signature will have only two values: 0 and 1.

Tab. 3.8 Survival signature and DPLDs for storage system

Type 1 (l_1)	Type 2 (l_2)	Type 3 (l_3)	Type 4 (l_4)	$\Phi(l_1, l_2, l_3, l_4)$	$\frac{\partial\Phi\downarrow}{\partial l_1\downarrow}$	$\frac{\partial\Phi\downarrow}{\partial l_2\downarrow}$	$\frac{\partial\Phi\downarrow}{\partial l_3\downarrow}$	$\frac{\partial\Phi\downarrow}{\partial l_4\downarrow}$
0	0	0	0	0	-	-	-	-
0	0	0	1	0	-	-	-	0
0	0	1	0	0	-	-	0	-
0	0	1	1	1	-	-	1	1
0	1	0	0	1	-	1	-	-
0	1	0	1	1	-	1	-	0
0	1	1	0	1	-	1	0	-
0	1	1	1	1	-	0	0	0
1	0	0	0	1	1	-	-	-
1	0	0	1	1	1	-	-	0
1	0	1	0	1	1	-	0	-
1	0	1	1	1	0	-	0	0
1	1	0	0	1	0	0	-	-
1	1	0	1	1	0	0	-	0
1	1	1	0	1	0	0	0	-
1	1	1	1	1	0	0	0	0
SI_k					0.375	0.375	0.125	0.125

Based on these facts, we will focus our analysis on same system, but with different types of HDDs, more precisely with two different types of HDDs. HDDs 1 and 3 will be SEAGATE ST6000DX000 and HDDs 2 and for will be WDC WD60EFRX. By taking this changes into account, the survival signature will be different from the previous one shown in Tab. 3.8 and its values can be seen in Tab. 3.9.

Tab. 3.9 Survival signature for storage system with two types of HDDs

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1, l_2)$
0	0	0
0	1	0.5
0	2	1
1	0	0.5
1	1	1
1	2	1
2	0	1
2	1	1
2	2	1

From this survival signature we can continue in our reliability analysis by computing all new DPLDs for each type of system components. The first DPLD and $\mathrm{SI}_{k,a}^{\downarrow}$ for each type is shown in Tab. 3.10. From values of the first DPLDs and $\mathrm{SI}_{k,a}^{\downarrow}$ it is possible to see, that they are symmetrical for both types. This comes from the fact, that both types are distributed evenly in terms of system topology. For example, the DPLD $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(1\to 0)}$ for l_1 has value 1 for $l_2 \in \{0,1\}$ as the $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(1\to 0)}$ for l_2 when $l_1 \in \{0,1\}$. This value of $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(1\to 0)}$ indicates the decrease of the system survivability represented by system signature if number of working components of type 1 decrease from one working components to zero, with zero or one working components of type 2. This is understandable according to the system topology, because if component of type 1 fails when one component of type 2 is working, then the system can lose one path for data reading and writing (either if HDD 1 fails or HDD 3 fails if HDD4 is working) and if none component of type 2 is working, then the system will surely fail if it is not failed already.

Tab. 3.10 The first DPLDs for storage system with two types of HDDs

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1,l_2)$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(2\to 1)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(1\to 0)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(2\to 1)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(1\to 0)}$
0	0	0	-	-	-	-
0	1	0.5	-	-	-	1
0	2	1	-	-	1	-
1	0	0.5	-	1	-	-
1	1	1	-	1	-	1
1	2	1	-	0	0	-
2	0	1	1	-	-	-
2	1	1	0	-	-	0
2	2	1	0	-	0	-
$\mathrm{SI}_{k,a}^{\downarrow}$			0.333	0.667	0.333	0.667

By using the first type of DPLDs and $\mathrm{SI}_{k,a}^{\downarrow}$, we can compute the second type of DPLDs according to (2.14) and $\mathrm{SI}_{k}^{\downarrow}$ according to (2.15) and their values can be seen in Tab. 3.11. From this DPLD we can see the type influence for the system state more clearly. For example, if we take the DPLD $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}$ for type 1 we can see that in half cases the failure of the component of type 1 affects the system survivability represented by survival signature as in case of DPLD $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2\downarrow}$. For $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}$ it comes in the situations in which there is none component of type 2 working or there is one component of type 1 and 2 working. In those situations the system survivability will decrease or the system will surely fail.

Tab. 3.11 The second DPLDs for storage system with two types of HDDs

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1, l_2)$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2\downarrow}$
0	0	0	-	-
0	1	0.5	-	1
0	2	1	-	1
1	0	0.5	1	-
1	1	1	1	1
1	2	1	0	0
2	0	1	1	-
2	1	1	0	0
2	2	1	0	0
$\mathrm{SI}_k^{\downarrow}$			0.5	0.5

By using the second DPLD for each type, we can see where the system survivability changes with the decrease of the number of working components of type 1 or 2. But if we wanted to see this degradation and average degradation more precisely, then the third DPLD and $\mathrm{SI}_k^{\Downarrow}$ can be used and their values for each type can be seen in Tab. 3.12. This type of DPLD can help us to further understand, how critical is component failure of concrete type according to the actual number of working components of all types. For example, let us consider the DPLD $\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_1\downarrow}$ for type 1. From it we can see, that all the significant changes decreases the system survivability evenly by 0.5. This means that if any component of type 1 fails, then the system survivability will decrease and in one situation the system will fail.

Tab. 3.12 The third DPLDs for storage system with two types of HDDs

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1, l_2)$	$\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_1\downarrow}$	$\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_2\downarrow}$
0	0	0	-	-
0	1	0.5	-	0.5
0	2	1	-	0.5
1	0	0.5	0.5	-
1	1	1	0.5	0.5
1	2	1	0	0
2	0	1	0.5	-
2	1	1	0	0
2	2	1	0	0
$\mathrm{SI}_k^{\Downarrow}$			0.25	0.25

3.2.3 Case Study for System with Bridge Topology

We will demonstrate the computation of the Survival Signature on the system that is described in [18] and its reliability block diagram is shown in Fig. 3.25. This specific system is composed of six components and we will represent their states by using Boolean variables $x_1, x_2, x_3, x_4, x_5, x_6$. There are two different types of system components, especially components represented by Boolean variables x_1, x_2, x_3 have same type (type1) and x_4, x_5, x_6 have same type (type 2). This is shown with label in top left corner and different colours in Reliability Block diagram in Fig. 3.25 (green – type 1 and blue – type 2). The structure function representing this system has following form:

$$\phi(\boldsymbol{x}) = x_1 \wedge (x_2 \wedge x_3 \vee x_2 \wedge x_4 \wedge x_6 \vee x_5 \wedge x_6 \vee x_5 \wedge x_4 \wedge x_3). \qquad (3.28)$$

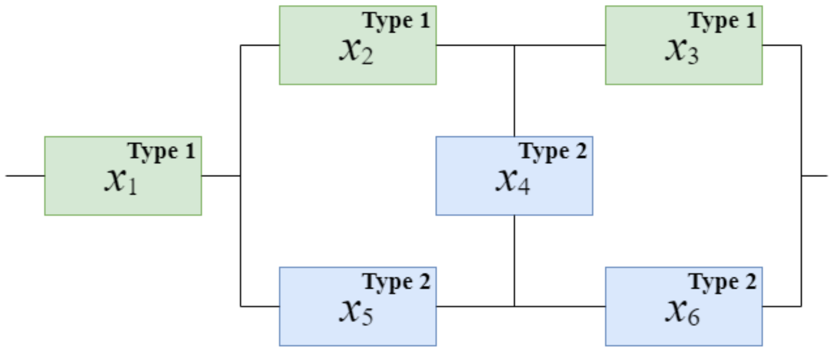


Fig. 3.25 Reliability block diagram for the analysed system with bridge topology

The survival signature for system can be seen in Tab. 3.13. The next step is computation of all new DPLDs for each type of system components. The first DPLD and $\mathrm{SI}_{k,a}^{\downarrow}$ for each type is shown in Tab. 3.14. For example, the DPLD $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(3\to2)}$ for l_1 has value 1 for each $l_2 \in (3,2,1,0)$. This means that if number of working components of type 1 decrease from three working components to two, with any number of working components of type 2 the system survivability represented by system signature will also decrease. This is understandable according to the system topology, because if component represented by variable x_1 of type 1 fails, then the system will surely fail. On the other hand, the DPLD $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(1\to0)}$ for l_2 has value 0 for each $l_1 \in (3,2,1,0)$. This means that if number of working components of type 2 decrease from one to none, the system survivability represents by the system signature will remain same with any number of working components of type 1. This is caused by fact, that if only one component of type 2 is working, then this type does not influence the state of the system. This importance is further proved by values of $\mathrm{SI}_{1,3}^{\downarrow}$ and $\mathrm{SI}_{2,1}^{\downarrow}$.

Tab. 3.13 Survival signature for analysed system represented by (3.28)

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1, l_2)$
0	0	0
0	1	0
0	2	0
0	3	0
1	0	0
1	1	0
1	2	0.11111
1	3	0.33333
2	0	0
2	1	0
2	2	0.44444
2	3	0.66667
3	0	1
3	1	1
3	2	1
3	3	1

Tab. 3.14 The first DPLDs for analysed system represented by (3.28)

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1, l_2)$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(3\to2)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(2\to1)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(1\to0)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(3\to2)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(2\to1)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(1\to0)}$
0	0	0	-	-	-	-	-	-
0	1	0	-	-	-	-	-	0
0	2	0	-	-	-	-	0	-
0	3	0	-	-	-	0	-	-
1	0	0	-	-	0	-	-	-
1	1	0	-	-	0	-	-	0
1	2	0.11111	-	-	1	-	1	-
1	3	0.33333	-	-	1	1	-	-
2	0	0	-	0	-	-	-	-
2	1	0	-	0	-	-	-	0
2	2	0.44444	-	1	-	-	1	-
2	3	0.66667	-	1	-	1	-	-
3	0	1	1	-	-	-	-	-
3	1	1	1	-	-	-	-	0
3	2	1	1	-	-	-	0	-
3	3	1	1	-	-	0	-	-
$\mathrm{SI}_{k,a}^{\downarrow}$			1	0.5	0.5	0.5	0.5	0

From the first type of DPLDs we can compute the second type of DPLDs according to (2.14) and we can compute $\mathrm{SI}_k^{\downarrow}$ from $\mathrm{SI}_{k,a}^{\downarrow}$ according to (2.15), which is shown in Tab. 3.15. From this DPLD we can see the type influence for the system state more clearly. For example, if we take the DPLD $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}$ for type 1 we can see, that in most cases the failure of the component of type 1 affects the system survivability represented by survival signature. Only exception are situations, in which two components of type 1 are working or one component of type 1 is working and one or none component of type 2 is working. In those cases, the system is in failed state, therefore the failure of component of type 1 does not have any influence on the system state. From values of $\mathrm{SI}_1^{\downarrow}$ and $\mathrm{SI}_2^{\downarrow}$ we can see that the decrease of the working components of type 1 is more significant for the system survivability.

Tab. 3.15 The second DPLDs for analysed system represented by (3.28)

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1,l_2)$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2\downarrow}$
0	0	0	-	-
0	1	0	-	0
0	2	0	-	0
0	3	0	-	0
1	0	0	0	-
1	1	0	0	0
1	2	0.11111	1	1
1	3	0.33333	1	1
2	0	0	0	-
2	1	0	0	0
2	2	0.44444	1	1
2	3	0.66667	1	1
3	0	1	1	-
3	1	1	1	0
3	2	1	1	0
3	3	1	1	0
$\mathrm{SI}_k^{\downarrow}$			0.75	0.25

If we wanted to see precisely how the survivability changes with the decrease of the working components of type 1 or 2, then the third DPLD and $\mathrm{SI}_k^{\Downarrow}$ can be used and their value for each type can be seen in Tab. 3.16. This type of DPLD and $\mathrm{SI}_k^{\Downarrow}$ can help us to further understand, how critical is component failure of concrete type according to the actual number of working components for all types and how critical is concrete type for system survivability. For example, let us consider the DPLD $\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_1\downarrow}$ for type 1. From it we can see, that the most critical failure of component of type 1 is when there are 3 working components of type 1 and one or none working component of type 2. In this situation, if any component of type 1 fails, then the system will surely fail.

Tab. 3.16 The third DPLDs for analysed system represented by (3.28)

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1, l_2)$	$\frac{\partial\Phi(l_1,l_2) \Downarrow}{\partial l_1 \downarrow}$	$\frac{\partial\Phi(l_1,l_2) \Downarrow}{\partial l_2 \downarrow}$
0	0	0	-	-
0	1	0	-	0
0	2	0	-	0
0	3	0	-	0
1	0	0	0	-
1	1	0	0	0
1	2	0.11111	0.11111	0.11111
1	3	0.33333	0.33333	0.22222
2	0	0	0	-
2	1	0	0	0
2	2	0.44444	0.33333	0.44444
2	3	0.66667	0.33333	0.22222
3	0	1	1	-
3	1	1	1	0
3	2	1	0.55556	0
3	3	1	0.33333	0
$\mathrm{SI}_k^{\Downarrow}$			0.25	0.083333

3.2.4 Case Study for Hydro Power Plant

Lastly, we will show how the proposed derivatives can be used in reliability analysis of the real world hydro power plant that is presented in [19]. We will be focusing on the inside mechanism of the hydro power plant, which can be seen in form of reliability block diagram in Fig. 3.26.

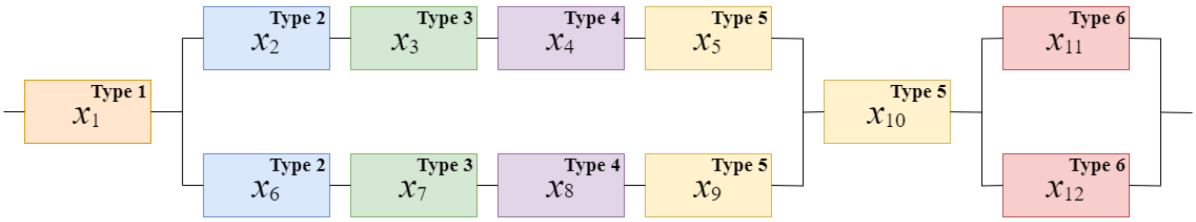


Fig. 3.26 Reliability block diagram for the hydro power plant

Firstly, the water comes from the reservoir through the gate (component x_1 of type 1) that controls the flow of the water to the two butterfly valves (components x_2 and x_6 of type 2). Then the water flows to the two turbines (components x_3 and x_7 of type 3) in which the kinetic energy of the water flow is used to move the turbine and to produce alternating current in the two generators (components x_4 and x_8 of type 4). Finally, there are three circuit breakers that protects the hydro power plant system (components x_5, x_9 and x_{10} of type 5) and two transformers (components x_{11} and x_{12} of type 6) used to obtain a higher voltage for the output electricity. The structure function representing this system has following form:

$$\phi(\boldsymbol{x}) = x_1 \wedge (x_2 \wedge x_3 \wedge x_4 \wedge x_5 \vee x_6 \wedge x_7 \wedge x_8 \wedge x_9) \wedge x_{10} \wedge (x_{11} \vee x_{12}). \qquad (3.29)$$

The non-zero values of the survival signature for system computed using (1.35) can be seen in Tab. 3.17. After obtaining the survival signature, we will compute all the DPLDs for each type. From the first, second DPLDs and $\mathrm{SI}_k^{\downarrow}$ (second DPLDs for the non-zero values of the survival signature and $\mathrm{SI}_k^{\downarrow}$ can be seen in Tab. 3.18) we can understand, that the most important type is type 1, which is caused by the fact, that this type has only one component and it's the first component in series topology.

In order to further understand, how critical is component failure of concrete type according to the actual number of working components for all types and how critical is concrete type for system survivability, the third type of DPLDs can be used and its values for each non-zero values of the system signatures that are shown in Tab. 3.17 and $\mathrm{SI}_k^{\Downarrow}$ values can be seen in Tab. 3.19. For example, let us consider the DPLD $\frac{\partial\Phi\Downarrow}{\partial l_5\downarrow}$ for type 5. This type is quite interesting, because according to the reliability block diagram in Fig. 3.26 the functionality of the component x_{10} of type 5 is crucial for the system functionality. This can be seen in situations, when only two components of type 5 is functioning and the system is functioning. In this situations, if one component of type 5 fails, then the system will surely fail. However, the most crucial type is type 1 which can be proved by the value of $\mathrm{SI}_k^{\Downarrow}$.

Tab. 3.17 Non-zero values of the survival signature for analysed hydro power plant

Type 1 (l_1)	Type 2 (l_2)	Type 3 (l_3)	Type 4 (l_4)	Type 5 (l_5)	Type 6 (l_6)	$\Phi(l_1, l_2, l_3, l_4, l_5, l_6)$
1	1	1	1	2	1	0.08333
1	1	1	1	2	2	0.08333
1	1	1	1	3	1	0.25
1	1	1	1	3	2	0.25
1	1	1	2	2	1	0.16667
1	1	1	2	2	2	0.16667
1	1	1	2	3	1	0.5
1	1	1	2	3	2	0.5
1	1	2	1	2	1	0.16667
1	1	2	1	2	2	0.16667
1	1	2	1	3	1	0.5
1	1	2	1	3	2	0.5
1	1	2	2	2	1	0.33333
1	1	2	2	2	2	0.33333
1	1	2	2	3	1	1
1	1	2	2	3	2	1
1	2	1	1	2	1	0.16667
1	2	1	1	2	2	0.16667
1	2	1	1	3	1	0.5
1	2	1	1	3	2	0.5
1	2	1	2	2	1	0.33333
1	2	1	2	2	2	0.33333
1	2	1	2	3	1	1
1	2	1	2	3	2	1
1	2	2	1	2	1	0.33333
1	2	2	1	2	2	0.33333
1	2	2	1	3	1	1
1	2	2	1	3	2	1
1	2	2	2	2	1	0.66667
1	2	2	2	2	2	0.66667
1	2	2	2	3	1	1
1	2	2	2	3	2	1

Tab. 3.18 The second DPLDs for the non-zero values of the survival signature of hydro power plant

	$\frac{\partial\Phi\downarrow}{\partial l_1\downarrow}$	$\frac{\partial\Phi\downarrow}{\partial l_2\downarrow}$	$\frac{\partial\Phi\downarrow}{\partial l_3\downarrow}$	$\frac{\partial\Phi\downarrow}{\partial l_4\downarrow}$	$\frac{\partial\Phi\downarrow}{\partial l_5\downarrow}$	$\frac{\partial\Phi\downarrow}{\partial l_6\downarrow}$
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	0	0	0	1	1
	1	0	0	0	1	0
$SI_k^{\downarrow}$	0.098765	0.069444	0.069444	0.069444	0.065843	0.037037

Tab. 3.19 The third DPLDs for the non-zero values of the survival signature of hydro power plant

	$\frac{\partial \Phi \Downarrow}{\partial l_1 \downarrow}$	$\frac{\partial \Phi \Downarrow}{\partial l_2 \downarrow}$	$\frac{\partial \Phi \Downarrow}{\partial l_3 \downarrow}$	$\frac{\partial \Phi \Downarrow}{\partial l_4 \downarrow}$	$\frac{\partial \Phi \Downarrow}{\partial l_5 \downarrow}$	$\frac{\partial \Phi \Downarrow}{\partial l_6 \downarrow}$
	0.08333	0.08333	0.08333	0.08333	0.08333	0.08333
	0.08333	0.08333	0.08333	0.08333	0.08333	0
	0.25	0.25	0.25	0.25	0.16667	0.25
	0.25	0.25	0.25	0.25	0.16667	0
	0.16667	0.16667	0.16667	0.08333	0.16667	0.16667
	0.16667	0.16667	0.16667	0.08333	0.16667	0
	0.5	0.5	0.5	0.25	0.33333	0.5
	0.5	0.5	0.5	0.25	0.33333	0
	0.16667	0.16667	0.08333	0.16667	0.16667	0.16667
	0.16667	0.16667	0.08333	0.16667	0.16667	0
	0.5	0.5	0.25	0.5	0.33333	0.5
	0.5	0.5	0.25	0.5	0.33333	0
	0.33333	0.33333	0.16667	0.16667	0.33333	0.33333
	0.33333	0.33333	0.16667	0.16667	0.33333	0
	1	1	0.5	0.5	0.66667	1
	1	1	0.5	0.5	0.66667	0
	0.16667	0.08333	0.16667	0.16667	0.16667	0.16667
	0.16667	0.08333	0.16667	0.16667	0.16667	0
	0.5	0.25	0.5	0.5	0.33333	0.5
	0.5	0.25	0.5	0.5	0.33333	0
	0.33333	0.16667	0.33333	0.16667	0.33333	0.33333
	0.33333	0.16667	0.33333	0.16667	0.33333	0
	1	0.5	1	0.5	0.66667	1
	1	0.5	1	0.5	0.66667	0
	0.33333	0.16667	0.16667	0.33333	0.33333	0.33333
	0.33333	0.16667	0.16667	0.33333	0.33333	0
	1	0.5	0.5	1	0.66667	1
	1	0.5	0.5	1	0.66667	0
	0.66667	0.33333	0.33333	0.33333	0.66667	0.66667
	0.66667	0.33333	0.33333	0.33333	0.66667	0
	1	0	0	0	0.33333	1
	1	0	0	0	0.33333	0
$\mathrm{SI}_k^{\Downarrow}$	0.049382	0.023148	0.023148	0.023148	0.023662	0.018518

Conclusion

The current state and level of technology brings new challenges in the development of the reliability engineering, which has to be able to deal with analysis of complex systems, i.e. systems composed of many components with various behaviour. Investigation of such systems requires development of new approaches that allow describing their properties in an efficient way and new methods that allow analysing their properties in a reasonable time. A possible solution to the first task is application of survival signature [15], which represents a compact form of system structure function. A solution to the second problem can be use of methodology of logic differential calculus, whose application in time-independent reliability analysis has been considered in [13], [14]. However, for real-world problems, it is very important to be able to perform time-dependent analysis, which allows us to find how properties of the system change as time flows. In this thesis, we showed that *logic differential calculus can also be used in time-dependent reliability analysis*, which expands possibilities of its application in solving real-world problems, and we proposed *the concept how the methodology of logic differential calculus can be combined with survival signature*, which allows expanding its applicability on analysis of various properties of systems composed of a huge amount of various components. To achieve these two principal results, we had to:

- investigate the theoretical basis of the reliability analysis based on structure function:
  - ✓ it was presented how the structure function can be used in reliability analysis and how methodologies like logic differential calculus, system signature, and survival signature can be used in reliability analysis based on the structure function (chapter 1);
- analyse approaches proposed in [13], [14] for computation of time-independent IMs based on logic differential calculus:
  - ✓ it was shown how logic differential calculus can be used in computation of reliability and structure IMs (sections 2.1.1 and 2.1.2);
- expand the approach for computation of time-independent IMs on calculation of time-dependent IMs:
  - ✓ it was shown how logic differential calculus can be used in computation of lifetime IMs, especially for BI (section 2.1.3), which is a base of many other lifetime IMs, such as CI,

  - ✓ usability of the new approach was demonstrated on three case studies dealing with storage system, drone fleet and surveillance system (section 3.1);
- define extensions of DPLDs for reliability analysis based on survival signature and present their meaning and application in reliability analysis:
  - ✓ three new types of DPLDs, which allow studying consequences of a failure of one or more same components for the system, were proposed (section 2.2),
  - ✓ three new SI measures, which allow quantifying importance of components of the same type for the system operation, that uses newly defined DPLDs were proposed (section 2.2),
  - ✓ usability of the new DPLDs and SIs proposed in this thesis was shown on four case studies dealing with series-parallel system, storage system, system with bridge topology and hydro power plant (section 3.2).

As mentioned above the key contribution of this thesis lies in proving that the methodology of logic differential calculus can also be used in time-dependent reliability analysis and that it can be combined with survival signature to analyse how specific type of components (not a specific component) influences system operation.

In this work, we primarily deals with IMs based on the concept of criticality (IMs such as SI, BI, or CI). However, there also exist other types of IMs, which are based on another concept known as a concept of minimal cut sets or minimal path sets. Typical example of such IMs is Fussell-Vesely's importance [14], which quantifies how a failure (repair) of a component contributes to a failure (functioning) of the system. Therefore, in the further work, we would like to focus on the possibility of application of logic differential calculus in time-dependent analysis based on minimal cut sets and minimal path sets and whether an approach combining minimal cut or path sets with time-dependent analysis based on logic differential calculus can also be applied in reliability analysis based on survival signature.

Resume

1. Úvod do predmetu výskumu

Teória spoľahlivosti je multidisciplinárny vedený obor, ktorý poskytuje metódy potrebné na kvantifikáciu spoľahlivosti systému, testovanie návrhu systému, analýzu systému, analýzu jeho komponentov atď. Hlavné výzvy teórie spoľahlivosti je možné zhrnúť nasledovne [1]:

- uplatňovanie teoretických vedomostí a matematických techník na zabránenie alebo zníženie pravdepodobnosti výskytu zlyhaní;
- identifikácia a riešenie príčin zlyhaní, ktoré sa vyskytujú v systéme napriek snahe im predchádzať;
- definovanie procesov, ktoré budú riešiť prípadné zlyhania, ak sa príčiny týchto zlyhaní nevyriešia;
- uplatňovanie metód na odhad spoľahlivosti nového návrhu systému a analýza údajov o spoľahlivosti systému.

Dôležitým krokom v hodnotení spoľahlivosti systému je vytvorenie jeho matematického vyjadrenia. Na základe [1], táto matematická reprezentácia musí umožniť preskúmať zlyhania systému, napr. mechanizmami na zistenie zlyhania a jeho dôsledkov; meranie spoľahlivosti systému; analyzovanie kritických stavov z hľadiska spoľahlivosti systému; vypracovanie spôsobu údržby systému, či diagnostika a prognóza porúch.

Najčastejšie používanou matematickou reprezentáciou systému v analýze spoľahlivosti je model, ktorý zohľadňuje dva dôležité stavy systému: zlyhanie a fungovanie systému. Tento matematický model je známy ako dvojstavový systém (BSS) a používa hlavne dvojhodnotovú logiku, ktorá bola zavedená ako jedna z prvých [2]–[4] a bude sa používať aj v tejto práci. Ďalšou používanou matematickou reprezentáciou systému je model zvaný viacstavový systém (MSS). Táto matematická reprezentácia môže pracovať s viac ako dvoma úrovňami prevádzky systému a používa sa na definovanie viacerých stavov fungovania systému a jeho komponenty, čo sa využíva pre vykonanie podrobnejšej analýzy spoľahlivosti systému a jeho komponentov [5], [6].

Na základe týchto matematických modelov existuje množstvo metód na vyhodnotenie spoľahlivosti a zlyhania systému. Všetky tieto metódy možno rozdeliť do štyroch skupín v závislosti od použitého matematického prístupu [1], [3]: metódy založené na štruktúrnej funkcii, stochastické metódy, metódy založené na Monte Carlo simulácii a metódy založené na univerzálne generovanej funkcii. Metódy založené na štruktúrnych funkciách umožňujú matematicky reprezentovať systém akejkoľvek štrukturálnej zložitosti [2] a budú použité v tejto práci.

Štruktúrna funkcia definuje ako závisí stav systému na stavoch jeho komponentov a používa sa na reprezentáciu systému zloženého z n komponentov [7]. Štruktúrnu funkciu možno považovať za booleovskú funkciu pre BSS ktorú možno použiť pri analýze spoľahlivosti systému [7], [8]. Takáto matematická reprezentácia je nezávislá od času. Dôležitou výhodou tejto reprezentácie je možnosť použiť rozvinutý a pre analýzu vhodný matematický prístup Boolovej algebry pri hodnotení spoľahlivosti skúmaného systému. Efektívne metódy v analýze spoľahlivosti boli vyvinuté s použitím Boolovej algebry na definíciu minimálnych množín rezov a ciest [8], frekvenčných charakteristík spoľahlivosti systému [7] alebo výpočtu indexov dôležitosti [9]. Štruktúrna funkcia má svoju dôležitú úlohu v modernom vývoji v analýze spoľahlivosti, napríklad v prípade viacfunkčnej spoľahlivosti systému [10], všeobecné vyjadrenie štruktúrnej funkcie ľubovoľného semi-koherentného systému [11] alebo grafové modely a algoritmy na hodnotenie spoľahlivosti [12]. Nevýhodou týchto metód je analýza systému bez riešenia času. Na druhej strane, štruktúrna funkcia vo forme booleovskej funkcie sa môže použiť na výpočet funkcie spoľahlivosti systému, ktorá predstavuje pravdepodobnosť, že systém bude vo funkčnom stave počas sledovaného obdobia alebo špecifického okamihu počas sledovaného obdobia. Hoci je v analýze spoľahlivosti dôležitá funkcia spoľahlivosti, neposkytuje úplný obraz o spoľahlivosti systému. Ďalším nevyhnutným prvkom hodnotenia spoľahlivosti je analýza dôležitosti. Metódy výpočtu indexov dôležitosti (IMs), ktoré kvantifikujú vplyv komponentov systému na celý systém reprezentovaný štruktúrnou funkciou za pomoci logického diferenciálneho počtu, boli predstavené v [13], [14], pričom nebrali do úvahy čas.

Štruktúrna funkcia môže byť vytvorená pre systém s akoukoľvek štrukturálnou zložitosťou. Súčasne sa so zvyšujúcim počtom komponentov systému zvyšuje aj dimenzia štruktúrnej funkcie, čo robí prácu s ňou pre veľké systémy viac náročnou z pamäťového a výpočtového hľadiska, hlavne pri nejednoznačnosti správania komponentov. Preto sa pre štruktúrnu funkciu vyvíjajú metód, ktoré by riešili tento problém. Jedným z moderných prístupov je použitie podpisu prežitia (angl. *Survival signature*) [15], ktorý sa zameriava na schopnosť prežitia systému s K typmi komponentov, pričom sa tento prístup dá použiť aj v časovo orientovanej analýze spoľahlivosti [16]. Tento prístup sa naďalej rozširuje [17], [18] a tiež v [19] je navrhnutý prístup pre analýzu dôležitosti systému. Táto metóda je účinná, ale je náročná na výpočty. Metódy analýzy dôležitosti sa zvyčajne zakladajú na rôznych matematických prístupov [14]. Jedným z takýchto prístupov je logický diferenciálny počet [21], [22], ktorý sa štandardne používa na analýzu systému vyjadreného štruktúrnou funkciou a nie na podpis prežitia.

Zohľadnením všetkých vyššie uvedených informácií je štruktúrna funkcia vo forme booleovskej funkcie matematické znázornenie systému používané v analýze spoľahlivosti, ktorá môže byť vytvorená pre systém akejkoľvek štrukturálnej zložitosti a umožňuje použitie metód používaných na booleovskej funkcie v analýze spoľahlivosti. Nevýhody tejto matematickej reprezentácie sú (a) vysoký rozmer pre systém s veľkým počtom komponentov a (b) nemožnosť časovo závislej analýzy. Hlavným cieľom tejto práce je vývoj nových prístupov pre analýzu spoľahlivosti systému založenej na štruktúrnej funkcii, ktoré umožnia časovo závislú analýzu systému a tiež ktoré riešia problém reprezentácie systému s veľkým počtom komponentov. Prvá časť tohto cieľa bude riešená použitím logického diferenciálneho počtu a druhá časť bude riešená pomocou podpisu prežitia pre reprezentáciu systému. Na dosiahnutie hlavného cieľa sú v tejto práci definované nasledujúce úlohy:

- nadviazanie na výskum z [13], [14] a navrhnutie prístupu v časovo závislej analýze dôležitosti systému, ktorý bude založený na logickom diferenciálnom počte v Boolovej algebre;
    - demonštrácia použitia navrhovaného prístupu na vybraných systémoch;
- definovanie parciálnych derivácií použiteľných v analýze spoľahlivosti systému založených na podpise prežitia systému [15];
    - demonštrácia použitia navrhovaného prístupu na vybraných systémoch.

2. Časovo závislá analýza dôležitosti s použitím logického diferenciálneho počtu

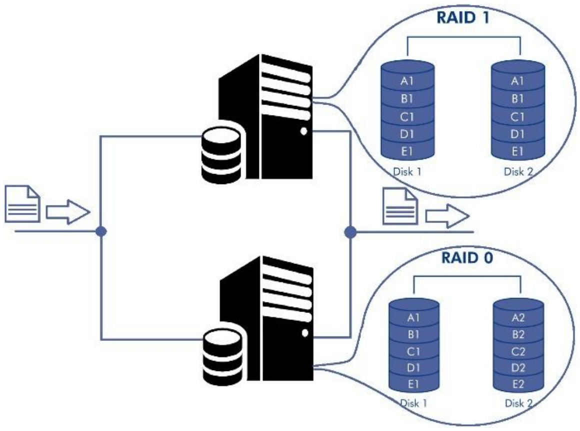
Pro časovo závislej analýze dôležitosti bude používaná štruktúrna funkcia ako matematický popis dvojstavového koherentného [3], [14] neopraviteľného systému. Štruktúrna funkcia je mapovanie, ktoré definuje hodnotu stavu systému pre každú kombináciu stavov n systémových komponentov, čo je definované nasledovne [3]:

$$\phi(x_1, x_2 \dots, x_n) = \phi(\boldsymbol{x}): \ \{0,1\}^n \rightarrow \{0,1\}, \tag{1}$$

kde x_i je booleovská premenná reprezentujúca stav komponentu i pre $i = 1,2, \dots, n$ a $\boldsymbol{x} = (x_1, x_2, \dots, x_n)$ je vektor stavov všetkých systémových komponentov (stavový vektor). Napríklad, pre dátové úložisko zobrazené na Obr. 1 by štruktúrna funkcia vyzerala nasledovne:

$$\phi(x_1, x_2, x_3, x_4) = (x_1 \vee x_2) \vee x_3 \wedge x_4. \tag{2}$$

Dôvodom je, že dané úložisko obsahuje dve úložne jednotky zapojené paralelne, čo môžeme reprezentovať Boolovou operáciou OR. Prvá jednotka má dva pevné disky (HDDs) zapojené v RAID 1, čo je možné reprezentovať Boolovou operáciou OR a druhá jednotka ma dva HDDs zapojené v RAID 0, čo je možné reprezentovať Boolovou operáciou AND. Tiež je potrebné podotknúť, že tento systém je koherentný (každý HDD má vplyv na funkčnosť systému a štruktúrna funkcia je neklesajúca pre každý HDD) a tiež sa bude pokladať za neopraviteľný.



Obr. 1 Dátové úložisko

Štruktúrna funkcia nájde svoje využitie aj pri skúmaní topologických vlastností systému [8], [13], [17], [23], [24]. Avšak nie je stavaná pre vykonanie časovo závislej analýzy spoľahlivosti, ktorá sa zaoberá vyhodnotením spoľahlivosti systému v priebehu času. Na tieto účely sa používa stavová funkcia systému $z(t)$, ktorá je definovaná nasledovne:

$$z(t) = \phi(\boldsymbol{x}(t)) = \phi(x_1(t), x_2(t), \dots, x_n(t)) : \langle 0, \infty) \rightarrow \{0,1\}, \tag{3}$$

kde $x_i(t)$ pre $i = 1,2, \dots, n$ je funkcia, ktorá definuje stav i-tého komponentu v čase t. Aj keď stavová funkcia systému $z(t)$ úzko súvisí so štruktúrnou funkciou $\phi(\boldsymbol{x})$, tieto dve funkcie sa svojou povahou veľmi líšia, pretože prvá je funkciou času, zatiaľ čo druhá je funkciou definujúcou topológiu systému, ktorá je nezávislá od času. V prípade dátového úložiska by stavová funkcia vyzerala nasledovne [3]:

$$\phi(x_1(t), x_2(t), x_3(t), x_4(t)) = (x_1(t) \vee x_2(t)) \vee x_3(t) \wedge x_4(t). \tag{4}$$

Funkciu stavu systému je možné chápať ako spojenie štruktúrnej funkcie systému a jednej konkrétnej realizácie stavových funkcií všetkých komponentov systému, čo znamená, že stavovú funkcia systému $z(t)$ možno tiež považovať za jednu realizáciu nespočetného množstva stavových funkcií systému. To znamená, že vývoj systému v priebehu času možno považovať za nasledujúci stochastický proces:

$$\{Z(t); t \geq 0\}, \tag{5}$$

kde $Z(t)$ je náhodná premenná modelujúca správanie systému v čase t.

Definujme $Z(t)$ vo fixnom čase. Získavame náhodnú premennú X, ktorá môže mať hodnoty 0 a 1 s pravdepodobnosťou A alebo U, ktoré predstavujú dostupnosť, resp. nedostupnosť systému [3] a pri neopraviteľných systémoch zodpovedajú spoľahlivosti R, resp. nespoľahlivosti F systému [3]. Pre jednotlivé komponenty systému sú to pravdepodobnosti p_i and q_i, ktoré sú definované nasledovne [3]:

$$\begin{gathered} p_i = \Pr\{x_i = 1\}, q_i = \Pr\{x_i = 0\}, \\ p_i + q_i = 1. \end{gathered} \tag{6}$$

Ak poznáme náhodnú premennú x_i, ktorá modeluje správanie sa komponentu i vo fixnom čase pre každú zložku systému, t.j. pre $i = 1,2, \dots, n$, a ak predpokladáme, že komponenty sú nezávislé, potom náhodnú premennú X možno získať kombináciou náhodných premenných x_i pomocou štruktúrnej funkcie. To umožňuje vypočítať pravdepodobnosti stavu systému nasledovne [3]:

$$p = \Pr\{\phi(\boldsymbol{x}) = 1\}, q = \Pr\{\phi(\boldsymbol{x}) = 0\}, \tag{7}$$

Na základe tejto definície je možné pokladať dostupnosť a nedostupnosť systému za funkciu pravdepodobností stavu komponentov [3]:

$$A = A(\boldsymbol{p}) = \Pr\{\phi(\boldsymbol{x}) = 1\}, U = U(\boldsymbol{q}) = \Pr\{\phi(\boldsymbol{x}) = 0\},$$
$$A + U = 1, \tag{8}$$

kde $\boldsymbol{p} = (p_1, p_2, \dots, p_n)$ a $\boldsymbol{q} = (q_1, q_2, \dots, q_n)$ sú vektory, ktorých prvky sú pravdepodobnosti stavov jednotlivých systémových komponentov. Toto môže byť použité na preskúmanie vplyvu špecifických zmien pravdepodobnosti stavu jedného alebo viacerých komponentov na pravdepodobnosti stavu systému alebo spoľahlivostné indexy [3], [14], avšak nie časovo závislú analýzu BSS. Pre časovo závislú analýzu je potrebné nahradiť náhodnú premennú X za $Z(t)$, ktorá definuje, ako sa menia vlastnosti náhodnej premennej X v čase. V tomto prípade je dostupnosť $A(t)$ a nedostupnosť $U(t)$ systému funkciou času, t.j.:

$$A(t) = A\big(\boldsymbol{P}(t)\big) = \Pr\big\{\phi\big(\boldsymbol{x}(t)\big) = 1\big\}, t \geq 0,$$
$$U(t) = U(\boldsymbol{Q}(t)) = \Pr\big\{\phi\big(\boldsymbol{x}(t)\big) = 0\big\}, t \geq 0, \tag{9}$$
$$A(t) + U(t) = 1, t \geq 0,$$

kde $\boldsymbol{P}(t) = \big(P_1(t), P_2(t), \dots, P_n(t)\big)$ a $\boldsymbol{Q}(t) = \big(Q_1(t), Q_2(t), \dots, Q_n(t)\big)$ sú vektorové funkcie, ktorých prvkami sú funkcie, ktoré definujú pravdepodobnosť stavu jednotlivých komponentov systému v priebehu času, a $\boldsymbol{x}(t) = (x_1(t), x_2(t), \dots, x_n(t))$ je vektor náhodných premenných, ktoré modelujú správanie sa komponentov systému v čase. To sa dá využiť na zistenie, ako sa mení spoľahlivosť systému alebo dôležitosť komponentov v čase.

Najdôležitejším prínosom predchádzajúcich vzorcov je to, že pravdepodobnosti stavu systému možno vnímať ako funkciu pravdepodobností stavov komponentov kombinovanú s použitím štruktúrnej funkcie alebo ako spojenie funkcií definujúcich pravdepodobnosti stavu komponentov systému v čase so štruktúrnou funkciou. To znamená, že ak sú systémové komponenty nezávislé a poznáme štruktúrnu funkciu systému a pravdepodobnosti stavov komponentov (v čase), dokážeme nájsť pravdepodobnosti stavov systému (v čase).

Definícia štruktúrnej funkcie zodpovedá definícii booleovskej funkcie [13], čo umožňuje použiť matematickú metodológiu booleovskej algebry v analýze spoľahlivosti založenej na štruktúrnej funkcii a jej časť známu ako logický diferenciálny počet, ktorá sa požíva napríklad na analýzu toho, ako zlyhanie komponentu ovplyvňuje fungovanie systému

[25]. Pokiaľ je potrebné analyzovať smer zmeny stavu komponentov, tak sa dá použiť priama smerová parciálna logická derivácia (DPLD) definovaná nasledovne [26]:

$$\frac{\partial\phi(1 \to 0)}{\partial x_i(1 \to 0)} = \frac{\partial\phi(0 \to 1)}{\partial x_i(0 \to 1)} = \overline{\phi(0_i, \boldsymbol{x})} \wedge \phi(1_i, \boldsymbol{x}), \tag{10}$$

kde $\wedge$ predstavuje Boolovu operáciu AND a $\overline{}$ je Boolova operácia NOT. Tu je potrebné podotknúť, že získaná DPLD je opäť booleovská funkcia. V prípade dátového úložiska by táto derivácia pre HDD 1 vyzerala nasledovne:

$$\begin{aligned}\frac{\partial\phi(1 \to 0)}{\partial x_1(1 \to 0)} &= \overline{\big((0 \vee x_2) \vee x_3 \wedge x_4)\big)} \wedge \big((1 \vee x_2) \vee x_3 \wedge x_4\big) \\ &= \overline{(x_2 \vee x_3 \wedge x_4)} \wedge 1 = \bar{x}_2 \wedge (\bar{x}_3 \vee \bar{x}_4).\end{aligned} \tag{11}$$

Táto DPLD naznačuje, že HDD 1 je pre systém kritický, t. j. jeho zlyhanie má za následok zlyhanie systému, ak zlyhá HDD 2 a aspoň jeden HDD z HDD 3 a 4. DPLD sa môže počítať nielen vzhľadom na zmenu stavu jednej zložky, ale aj vzhľadom na súčasnú zmenu stavu dvoch alebo viacerých komponentov, čo je smerová parciálna logická derivácia vypočítaná vzhľadom na vektor zmien a je definovaná nasledovne [8]:

$$\frac{\partial\phi(1 \to 0)}{\partial(x_i, x_j, \ldots)(1,1,\ldots) \to (0,0,\ldots)} = \overline{\phi\big(0_i, 0_j, \ldots, \boldsymbol{x}\big)} \wedge \phi\big(1_i, 1_j, \ldots, \boldsymbol{x}\big). \tag{12}$$

Toto je možné využiť na analýzu toho istého smeru zmien stavov komponentov a systému, ale aj opačných zmien a dokonca aj rôznych zmien. Napríklad pri zlyhaní HDD 1 a HDD 2 v dátovom úložisku by táto smerová parciálna derivácia vyzerala nasledovne:

$$\frac{\partial\phi(1 \to 0)}{\partial(x_1, x_2)((1,1) \to (0,0))} = \overline{x_3} \vee \overline{x_4}, \tag{13}$$

čo znamená, že súčasné zlyhanie HDD 1 a 2 vedie k zlyhaniu systému, ak dôjde k zlyhaniu aspoň jedného HDD z HDD 3 a 4.

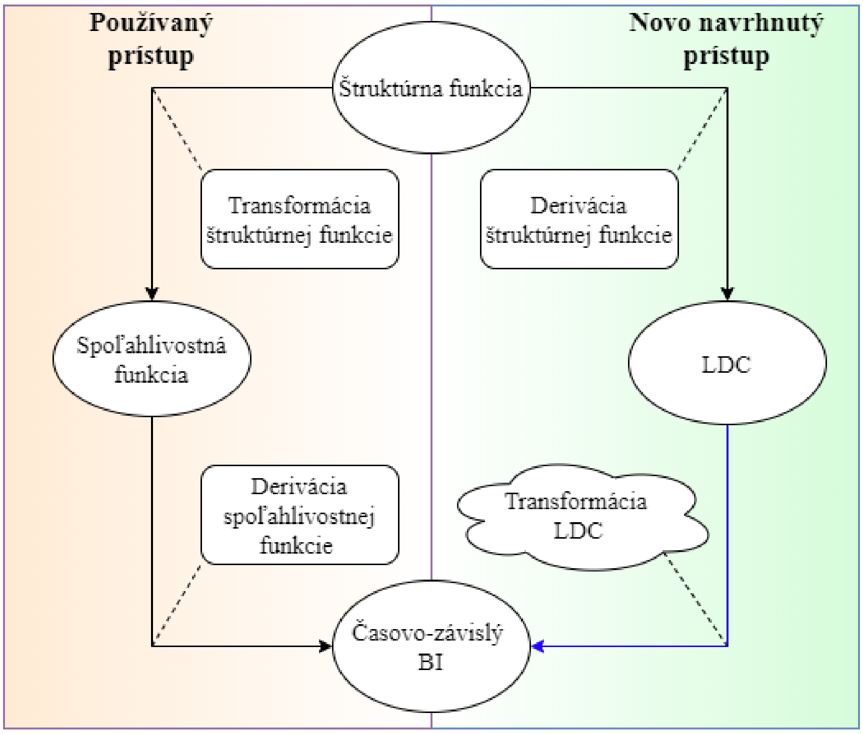
V analýze spoľahlivosti sa všetky uvedené smerové parciálne derivácie používajú napríklad na nájdenie kritických stavov systému [8], [13], ktoré opisujú situácie, v ktorých zlyhanie, resp. oprava jedného alebo viacerých komponentov systému má za následok zlyhanie, resp. opravu systému, alebo pri výpočte IM, ktoré je potom možné použiť na optimalizáciu spoľahlivosti systému, hľadania kritických komponentov systému, či pri plánovaní údržby systému. Existuje mnoho IMs a každý z nich berie do úvahy rôzne faktory, vďaka ktorým je komponent systému dôležitejší ako ostatné a delia sa do troch kategórií

[14]: štruktúrne (structural, S), spoľahlivostné (reliability, R) a časovo závislé (lifetime, L) IMs. Tabuľka 1 obsahuje prehľad vybraných IMs, pričom tiež uvádza vzorec pre štandardný výpočet daného IM a tiež výpočet založený na smerových parciálnych deriváciách. TD pri SI znamená hustota pravdy (angl. *truth density*) argumentu, ktorý je booleovskou funkciou.

Tabuľka 1 Prehľad vybraných IMs

Názov	**Typ**	**Popis**	**Štandardný výpočet**	**Výpočet s DPLD**
Štruktúrny index (SI)	S	relatívny počet stavových vektorov, pri ktorých zlyhanie komponentu i vedie k zlyhaniu systému	$\frac{\sum_{\{(0_i,x)\}}(\phi(1_i,x)-\phi(0_i,x))}{2^{n-1}}$	$\mathrm{TD}\left(\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}\right)$
Birnbaumov index (BI)	R	Pravdepodobnosť, že zlyhanie komponentu i spôsobí zlyhanie systému v definovanom čase	$\frac{\partial R}{\partial p_i}$	$\mathrm{Pr}\left\{\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}=1\right\}$
Kritický index (CI)	R	Pravdepodobnosť, že zlyhanie systému bolo spôsobené zlyhaním komponentu i ak systém zlyhal	$\mathrm{CI}_i=\mathrm{BI}_i\frac{q_i}{F}$	$\mathrm{CI}_i=\mathrm{BI}_i\frac{q_i}{F}$
Časovo závislý Birnbaumov index	L	Pravdepodobnosť, že systém je v stave, v ktorom je komponent i rozhodujúci pre fungovanie systému v čase t	$\mathrm{BI}_i(t)=\frac{\partial R(t)}{\partial P_i(t)}$	$\mathrm{Pr}\left\{\frac{\partial z(1\to 0,t)}{\partial x_i(1\to 0,t)}=1\right\}$
Časovo závislý kritický index	L	Pravdepodobnosť, že komponent i zlyhal v čase t a tento komponent je pre systém v čase t kritický ak systém už nie je funkčný v čase t	$\mathrm{CI}_i(t)=\mathrm{BI}_i(t)\frac{Q_i(t)}{F(t)}$	$\mathrm{CI}_i(t)=\mathrm{BI}_i(t)\frac{Q_i(t)}{F(t)}$

Tu je potrebné uviesť, že v tejto práci bol zavedený výpočet časovo závislého BI za použitia DPLD. BI bol zvolený hlavne preto, že predstavuje základ pre množstvo ďalších IMs, pričom jedným z nich je aj CI [14]. Obr. 2 ilustruje oba spôsoby výpočtu časovo závislého BI, pričom na ľavej strane sa nachádza štandardný prístup, v ktorom zo štruktúrnej funkcie získame spoľahlivostnú funkciu $R(t)$, ktorá pri neopraviteľných systémoch zodpovedá funkcie dostupnosti $A(t)$ a následne parciálne derivuje túto funkciu podľa $P_i(t)$. Novo navrhnutý spôsob tento princíp otáča a to tak, že najskôr sa vykoná smerová parciálna derivácia štruktúrnej funkcie podľa x_i, čím sa získa booleovská funkcia, ktorá popisuje situácie, v ktorých zlyhanie komponentu i spôsobí zlyhanie systému. Tú je potom možné použiť pre získanie pravdepodobnosti, že systém je v stave, v ktorom je komponent i rozhodujúci pre fungovanie systému v čase t použitím (9), čím získame časovo závislý BI.



Obr. 2 Spôsoby výpočtu časovo závislého BI

V prípade dátového úložiska by to znamenalo, že miesto toho, aby sa z (2) získala spoľahlivostná funkcia $R(t)$, ktorá má tvar:

$$R(t) = P_1(t) + P_2(t) - P_1(t)P_2(t) + P_3(t)P_4(t) - P_1(t)P_3(t)P_4(t) - P_2(t)P_3(t)P_4(t) + P_1(t)P_2(t)P_3(t)P_4(t), \tag{14}$$

a následne sa táto funkcia parciálne derivovala napríklad podľa $P_i(t)$ pre HDD1, sa parciálne zderivuje najskôr (2) podľa x_1 a pre výsledok (11) sa už získa časovo závislý BI použitím (9) a teda sa dostane časovo závislý BI pre HDD1, ktorý má tvar:

$$\mathrm{BI}_1(t) = 1 - P_2(t) - P_3(t)P_4(t) + P_2(t)P_3(t)P_4(t). \tag{15}$$

V práci sa tiež nachádzajú 3 prípadové štúdie, v ktorých je ukázané využitie tohto prístupu pre dátové úložisko, dronovú letku a strážny sledovací systém.

3. Podpis prežitia systému s použitím logického diferenciálneho počtu

Štruktúrna funkcia predstavuje užitočný a elegantný opis návrhu systému, má však určité obmedzenia. Napríklad v prípade porovnania návrhu systému je použitie tohto prístupu náročnejšie, najmä pri väčšom počte komponentov systému [1], [16]. Z tohto dôvodu vzniklo viacero matematických prístupov, pričom jeden z nich je známy ako podpis prežitia systému [15]. Tento prístup je možné použiť pre systémy s $K \geq 1$ typmi komponentov v časovo závislej a aj nezávislej analýze spoľahlivosti systémov. Podpis prežitia $\Phi(l_1, l_2, \dots, l_K), l_k = 0,1, \dots, n_k$ je definovaný ako pravdepodobnosť, že systém s n komponentmi je funkčný ak presne l_k systémových komponentov typu k je funkčných pre $k = 1,2, \dots, K$ a má nasledovný tvar:

$$\Phi(l_1, l_2, \dots, l_K) = \left[\prod_{k=1}^{K} \binom{n_k}{l_k}^{-1}\right] * \sum_{\boldsymbol{x} \in S_{l_1, l_2, \dots, l_K}} \phi(\boldsymbol{x}), \tag{16}$$

kde $S_{l_1, l_2, \dots, l_K}$ je množina všetkých stavových vektorov $\boldsymbol{x}$ s presne $l_1, l_2, \dots, l_K$ pracujúcimi systémovými komponentmi. V prípade dátového úložiska sa zavedie nasledovný predpoklad: HDD 1 a 3 sú typu 1 a HDD 2 a 4 sú typu 2. Tabuľka 2 obsahuje všetky hodnoty podpisu prežitia dátového úložiska. Z týchto hodnôt je zjavné, že systém má najhoršiu pravdepodobnosť prežitia v situáciách, kedy funguje len jeden HDD. Tiež je možné si všimnúť, že podpis prežitia systému predstavuje popis MSS (v ojedinelých prípadoch BSS), ktorý ale vzniká na základe typu komponentov z BSS, čo je potom využiteľné pri definovaní nových smerových parciálnych derivácií pre podpis prežitia.

Tabuľka 2 Podpis prežitia dátového úložiska

Typ 1 (l_1)	Typ 2 (l_2)	$\Phi(l_1, l_2)$
0	0	0
0	1	0.5
0	2	1
1	0	0.5
1	1	1
1	2	1
2	0	1
2	1	1
2	2	1

Keďže bolo v predošlej časti ukázané, ako je možné využiť DPLD pri výpočtoch IMs, tak by bolo vhodné použiť DPLD aj pri podpise prežitia, čím rozšírime jeho použiteľnosť pre analýzu spoľahlivosti. Z toho dôvodu sú v tejto práci definované tri nové DPLDs pre podpis prežitia.

Prvá DPLD pre podpis prežitia indikuje možnosť zlyhania systému pre daný počet funkčných komponentov daného typu, ak jeden z komponentov tohto typu zlyhá:

$$\frac{\partial\Phi(l_1,\dots,l_K)\downarrow}{\partial l_k(a\to a-1)} = \begin{cases} 1, & \Phi(l_1,\dots,a_k,\dots,l_K) > \Phi(l_1,\dots,a_k-1,\dots,l_K) \\ 0, & \text{inak} \end{cases} \quad (17)$$

kde $a \in \{1,2,\dots,n_k\}$ je počet pracujúcich komponentov typu $k \in \{1,2,\dots,K\}$. Táto parciálna derivácia je definovaná len pre $(l_1,\dots,a_k,\dots,l_K)$ pracujúcich komponentov každého typu a je nenulová iba ak $\Phi(l_1,\dots,a_k,\dots,l_K) > \Phi(l_1,\dots,a_k-1,\dots,l_K)$. Na základe toho, že podpis prežitia je možné chápať ako MSS, je možné túto deriváciu chápať aj ako integrovanú smerovú deriváciu deriváciu typu 2, ktorá je popísaná v [31]. Tiež je možné definovať SI využívajúci túto DPLD, ktorý je nasledovný:

$$\mathrm{SI}_{k,a}^{\downarrow} = \mathrm{TD}\left(\frac{\partial\Phi(l_1,\dots,l_K)\downarrow}{\partial l_k(a\to a-1)}\right), \quad (18)$$

Pričom táto definícia zodpovedá definícii $\mathrm{SI}_{i,s}^{\downarrow}$ v [31]. Tento SI predstavuje relatívny počet situácií, v ktorých je počet fungujúcich komponentov a typu k kritický pre degradáciu systému. Tabuľka 3 obsahuje všetky hodnoty prvej DPLD a tiež SI pre dátové úložisko. Z týchto hodnôt je možné usúdiť, že DPLD a SI sú symetrické pre oba typy. Toto je spôsobené tým, že oba typy sú rovnako rozmiestnené v topológii systému. Z hodnôt SI je možné tiež vidieť, že viac dôležité je zlyhanie komponentu typu 1 (resp. 2) vtedy, keď je funkčný už len jeden komponent daného typu.

Tabuľka 3 Prvá DPLD a SI pre dátové úložisko

Typ 1 (l_1)	Typ 2 (l_2)	$\Phi(l_1,l_2)$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(2\to1)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(1\to0)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(2\to1)}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(1\to0)}$
0	0	0	-	-	-	-
0	1	0.5	-	-	-	1
0	2	1	-	-	1	-
1	0	0.5	-	1	-	-
1	1	1	-	1	-	1
1	2	1	-	0	0	-
2	0	1	1	-	-	-
2	1	1	0	-	-	0
2	2	1	0	-	0	-
$\mathrm{SI}_{k,a}^{\downarrow}$			0.333	0.667	0.333	0.667

Druhá DPLD pre podpis prežitia naznačuje možnosť zlyhania systému, ak jeden zo systémových komponentov daného typu zlyhá:

$$\frac{\partial\Phi(l_1,\dots,l_K)\downarrow}{\partial l_k\downarrow}=\begin{cases}1, & \Phi(l_1,\dots,l_k,\dots,l_K)>\Phi(l_1,\dots,\widetilde{l_k},\dots,l_K)\\ 0, & \text{inak}\end{cases} \tag{19}$$

alebo

$$\frac{\partial\Phi(l_1,\dots,l_K)\downarrow}{\partial l_k\downarrow}=\bigcup_{a=1}^{n_k}\frac{\partial\Phi(l_1,\dots,l_K)\downarrow}{\partial l_k(a\to a-1)} \tag{20}$$

kde $\widetilde{l_k}=l_k-1$. Z definície (20) je zrejmé, že táto derivácia je zjednotením (17) pre každé $a=1,2,\dots,n_k$. Druhú DPLD je možné použiť pre výpočet SI nasledovne:

$$\mathrm{SI}_k^{\downarrow}=\mathrm{TD}\left(\frac{\partial\Phi(l_1,\dots,l_K)\downarrow}{\partial l_k\downarrow}\right)=\frac{\sum_{a=1}^{n_k}\mathrm{SI}_{k,a}^{\downarrow}}{n_k}, \tag{21}$$

pričom táto definícia zodpovedá definícii $\mathrm{SI}_i^{\downarrow}$ v [31]. Tento SI predstavuje relatívny počet situácií, v ktorých degradácia komponentu typu k spôsobí degradáciu systému. Tabuľka 4 obsahuje všetky hodnoty druhej DPLD a tiež SI pre dátové úložisko. Z týchto hodnôt a hlavne SI je ešte viac zrejmá symetrickosť typov komponentov. Táto derivácia a SI ponúkajú jasnejší pohľad na vplyv počtu pracujúcich komponentov daného typu na prežitie systému.

Tabuľka 4 Druhá DPLD a SI pre dátové úložisko

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1,l_2)$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}$	$\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2\downarrow}$
0	0	0	-	-
0	1	0.5	-	1
0	2	1	-	1
1	0	0.5	1	-
1	1	1	1	1
1	2	1	0	0
2	0	1	1	-
2	1	1	0	0
2	2	1	0	0
$\mathrm{SI}_k^{\downarrow}$			0.5	0.5

Tretia a posledná DPLD pre podpis prežitia ukazuje mieru zlyhania systému, ak zlyhá jeden z komponentov daného typu:

$$\frac{\partial\Phi(l_1,\dots,l_K)\Downarrow}{\partial l_k\downarrow}=\begin{cases}\xi, & \Phi(l_1,\dots,l_k,\dots,l_K)>\Phi(l_1,\dots,\widetilde{l_k},\dots,l_K)\\ 0, & \text{inak}\end{cases} \tag{22}$$

kde $\xi=\Phi(l_1,\dots,l_k,\dots,l_K)-\Phi(l_1,\dots,\widetilde{l_k},\dots,l_K)$ pre l_k = 1, 2, .., n_k, $l_k>\widetilde{l_k}$ a $\widetilde{l_k}=l_k-1$. Ďalšia definícia tejto DPLD je nasledovná:

$$\frac{\partial\Phi(l_1,\dots,l_K)\Downarrow}{\partial l_k\downarrow} = (n_k)^{-1}\cdot\sum_{x_i\in N_k}\Phi\left(\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}\right) \tag{23}$$

kde $\Phi\left(\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}\right)$ transformácia každej $\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}$ na základe podpisu prežitia a N_k je množina všetkých komponentov typu k. SI pre túto deriváciu predstavuje priemerný úpadok hodnoty podpisu prežitia za predpokladu zníženia počtu pracujúcich komponentov typu k čo sa dá vyjadriť nasledovne:

$$\mathrm{SI}_k^{\Downarrow} = \frac{\sum_{\boldsymbol{l}\in S_k}\frac{\partial\Phi(\boldsymbol{l})\Downarrow}{\partial l_k\downarrow}}{n_k * \prod_{i\in M_k}(n_i+1)}, \tag{24}$$

kde $\boldsymbol{l} = (l_1,\dots,l_K)$ je vektor premenných, ktoré reprezentujú počet funkčných komponentov každého typu, S_k je množina všetkých vektorov $\boldsymbol{l}$, pre ktoré $l_k \in \{1,2,..,n_k\}$ a $l_i \in \{0,1,2,..,n_i\}$ pre $i = 1,\dots,k-1,k+1,\dots,K$ a M_k je množina $\{1,\dots,k-1,k+1,\dots,K\}$. Tabuľka 5 obsahuje všetky hodnoty tretej DPLD a tiež SI pre dátové úložisko. Táto DPLD a SI vyjadrujú ešte jasnejší pohľad na dôležitosť typov komponentov pre fungovanie systému.

Tabuľka 5 Tretia DPLD a SI pre dátové úložisko

Type 1 (l_1)	Type 2 (l_2)	$\Phi(l_1,l_2)$	$\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_1\downarrow}$	$\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_2\downarrow}$
0	0	0	-	-
0	1	0.5	-	0.5
0	2	1	-	0.5
1	0	0.5	0.5	-
1	1	1	0.5	0.5
1	2	1	0	0
2	0	1	0.5	-
2	1	1	0	0
2	2	1	0	0
$\mathrm{SI}_k^{\Downarrow}$			0.25	0.25

V práci sa tiež nachádzajú 4 prípadové štúdie, v ktorých je ukázané využitie tohto prístupu pre dátové úložisko, sériovo-paralelný systém, systém s mostovým zapojením a vodnú elektráreň.

4. Záver

Súčasný stav a vývoj technológií prináša nové výzvy v teórii spoľahlivosti, ako analýza zložitých systémov zložených z mnohých komponentov s rôznym správaním. Skúmanie takýchto systémov vyžaduje vývoj nových prístupov, ktoré umožňujú vhodne popísať ich vlastnosti, a tiež nových metód, ktoré umožňujú analýzu ich vlastností. Možným riešením prvej úlohy je použitie podpisu prežitia [15], ktorý predstavuje kompaktnú formu štruktúrnej funkcie systému. Riešením druhej úlohy môže byť použitie metodiky logického diferenciálneho počtu, ktorého použitie v časovo nezávislej analýze spoľahlivosti bolo ukázané v [13], [14]. Avšak pre riešenie problémov reálneho sveta je veľmi dôležité, aby sme boli schopní vykonať časovo závislú analýzu, ktorá nám umožňuje zistiť, ako sa menia vlastnosti systému v priebehu času. V tejto práci bolo ukázané, že logický diferenciálny počet sa dá použiť aj v časovo závislej analýze spoľahlivosti, ktorá rozširuje možnosti jeho aplikácie pri riešení problémov v reálnom svete, a tiež bol navrhnutý koncept, ako je možné využiť metodiku logického diferenciálneho počtu pre podpis prežitia, ktorý umožňuje rozšíriť jeho uplatniteľnosť na analýzu vlastností systémov zložených z veľkého množstva komponentov rôznych typov. Pre dosiahnutie týchto výsledkov bolo potrebné:

- preskúmať teoretické základy analýzy spoľahlivosti založenej na štruktúrnej funkcii:
    - ✓ bolo ukázané, ako je možné použiť štruktúrnu funkciu v analýze spoľahlivosti a ako je možné použiť metodológie ako logický diferenciálny počet, podpis systému a podpis prežitia, v analýze spoľahlivosti založenej na štruktúrnej funkcii;
- analyzovať prístupy navrhnuté v [13], [14] na výpočet časovo nezávislých IMs na základe logického diferenciálneho počtu:
    - ✓ bolo ukázané, ako sa dá logický diferenciálny počet použiť pri výpočte spoľahlivostných a štruktúrnych IMs;
- rozšíriť prístup pre výpočet IM nezávislých na čase pri výpočte časovo závislých IMs:
    - ✓ bolo ukázané, ako sa dá logický diferenciálny počet použiť pri výpočte časovo závislých IMs, najmä pre BI, ktorý je základom mnohých ďalších časovo závislých IMs, ako napríklad CI,

  - ✓ použiteľnosť nového prístupu bola demonštrovaná na troch prípadových štúdiách týkajúcich sa dátového úložiska, flotily dronov a strážneho sledovacieho systému;
- definovať DPLD pre analýzu spoľahlivosti založenej na podpise prežitia a ukázať ich význam a použitie v analýze spoľahlivosti:
  - ✓ boli navrhnuté tri nové typy DPLD, ktorými je možné skúmať následky zlyhania jedného alebo viacerých komponentov daného typu pre systém,
  - ✓ boli navrhnuté tri nové SIs, ktoré vyjadrujú dôležitosť komponentov daného typu pre fungovanie systému a používajú novo definované DPLD,
  - ✓ použiteľnosť nových DPLDs a SIs navrhnutých v tejto práci bola demonštrovaná na štyroch prípadových štúdiách zaoberajúcich sa sériovo-paralelným systémom, dátovým úložiskom, systémom s mostnou topológiou a vodnou elektrárňou.

Ako bolo uvedené vyššie, kľúčovým prínosom tejto práce je dokázanie, že metodika logického diferenciálneho počtu sa dá použiť aj v časovo závislej analýze spoľahlivosti a že ju možno kombinovať s podpisom prežitia na analýzu toho, ako daný typ komponentov (nie konkrétny komponent) ovplyvňuje fungovanie systému.

Táto práca sa zameriava predovšetkým na IMs založené na koncepte kritickosti (SI, BI alebo CI). Existujú však aj iné typy IMs, ktoré sú založené na inom koncepte známom ako koncept minimálnych rezov alebo minimálnych ciest. Typickým príkladom takýchto IMs je Fussell-Veselyho IM [14], ktorý vyjadruje, ako zlyhanie (oprava) komponentu prispieva k zlyhaniu (fungovaniu) systému. Preto bude budúci vývoj zameraný na možnosť použitia logického diferenciálneho počtu v časovo závislej analýze založenej na minimálnych rezoch a minimálnych cestách a na to, či prístup kombinujúci minimálne rezy alebo cesty s časovo závislou analýzou na základe logického diferenciálneho počtu možno použiť aj v analýze spoľahlivosti založenej na podpise prežitia.

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